INE PAN Working Paper Series

Paper number 57

Testable restrictions and identification of the preference discovery model

Marek Kapera

Warsaw, March 2024

wp@inepan.waw.pl

Testable restrictions and indentification of the preference discovery model

Marek Kapera*

March 2024

1 Introduction

Preference discovery hypothesis of Plott (1996) is a descriptive theory which states that the consumer discovers their taste only through consumption experience, meaning that before experiencing some alternatives, the consumer does not know what is their preference ranking with respect to these alternatives. However, in contrast to the similar theory of preference construction (for example Lichtenstein and Slovic 2006) the consumer has some real preferences which follow the typical assumptions of economic rationality that are discovered by consumption.

Empirical evidence, for example Kingsley and Brown (2010) or Czajkowski et al. (2015), show that consumer choices stabilize with market experience. It suggests that preference discovery takes place. Moreover, there is a lot of promising emipirical evidence (for example Cox and Grether 1996, Butler and Loomes 2007, Humphrey et al. 2017) which suggests that preference discovery hypothesis has a potential to explain a wide variety of observed paradoxes of choice, including WTA/WTP disparity and preference reversal. Results of van de Kuilen (2009) even suggest that probability weighting function introduced by prospect theory of Kahneman and Tversky (1979) converges to linearity as the consumer obtains market experience.

*Institute of Economics, Polish Academy of Sciences, 00-330 Warsaw, Nowy Swiat 72, Poland, kaperamar@gmail.com, +48 609 686 255, ORCID: 0000-0001-7028-5193

The study was funded by NCN Grant UMO-2020/37/N/HS4/03367.

At the same time, preference discovery has attracted limited theoretical interest (with a few exceptions like Loomes et al. 2009, Kapera 2022) and there are many studies sceptical towards this hypothesis, including Braga and Starmer (2005), Lichtenstein and Slovic (2006), Bruni and Sugden (2007), Braga et al. (2009). There are many reasons for this scepticism; preference discovery hypothesis as formulated by Plott (1996) is merely a descriptive theory without any formal model. Many elements of this hypothesis are imprecise, for example Plott assumes that if the consumer does not know their preferences, we might observe preference reversal because the consumer makes "mistakes" but it is absolutely not clear what those mistakes are and why should they occur. From the perspective of this article however, the most important is one of many critical comments with regard to this hypothesis by Bruni and Sugden (2007) that for preference discovery hypothesis to have any empirical significance, there must be some testable properties of this theory which hold across different knowledge sets of the consumer and it is not exactly clear what these restrictions might be.

In this article, I show that preference discovery admits restrictions that are testable across different knowledge sets of the consumer, and therefore cannot rationalize any arbitrary sequence of consumer choices. In my model of preference discovery I follow Kapera (2024) and assume that conditional preferences of the consumer, meaning the preference relations which dictate the consumer choices, can be represented by a subjective expected utility function. This representation follows the models of Kreps (1979), Cooke (2016) on the representation of preference for flexibility. The contribution of Cooke (2016) conditions the resolution of the uncertainty in the model of Kreps (1979) on experience from consumption.

My approach to the identification of testable restrictions of the model is loosely based on the contribution of Frick et al. (2019), where the authors identify testable restrictions of a dynamic random expected utility model. Dynamic random expected utility model, which is a generalization of the static random expected utility model of Gul and Pesendorfer (2006) to decision trees introduced by Kreps and Porteus (1978). Dynamic expected random utility models are closely connected to the preference discovery model of Kapera (2022, 2024), because in both models the consumer preference is probabilistic. The main difference is, that the probability has a different interpretation; in dynamic expected random utility the preference itself is random, whereas in the preference discovery model it is merely a representation for the lack of knowledge of their own preferences by the consumer and after the consumer experiences some alternatives, their preference for them becomes deterministic.

My results show, that due to observability problems, the typical individual demand data of the consumer is not enough for the existence of restrictions that are testable under different knowledge sets of the consumer. However in an extended setting with hypothetical choices and the addition of monetary payoffs to mimic option menus of Cooke (2016), the assumption that the consumer beliefs are evolve in a bayesian fashion is enough for the subjective expected utility model of the preference discovery to admit not only testable across different knowledge sets restrictions, but also to identify the beliefs of the consumer.

2 Behavioral model

The model of preference discovery follows the construction in Kapera (2024). Let (X, d) be a compact and separable metric space, equipped with a Borel measure λ. I assume, that λ is non-atomic, strictly positive on all open sets and that $\lambda(X) = 1$. Set X contains all of the possible alternatives, not all of which have to be available at each specific choice. Instead, each choice of the consumer is made from some smaller menu, which is a finite subset of X. Generic menu is denoted by m and M is the set of all possible menus, that is $M = \{m \subset X : \ |m| < \infty\}$.

The consumer is equipped with a preference relation s^*. However, this relation is ex-ante unknown and only partially revealed after the consumption. Ex ante, the only thing tat the consumer knows about s^* is that it is an element of set S, which is the set of all binary relations on X that satisfy axioms 1–3 defined below. Generic element of S is denoted by s and the relation of weak preference, strict preference and indifference with respect to preference $s \in S$ is denoted by $\succeq_s, \succ_s, \sim_s$ respectively.

Axiom 1. *(Rationality) Let $s \in S$. Then s is complete, reflexive and transitive.*

Axiom 2. *(Continuity) Let $s \in S$. For each $x \in X$ sets $\{y \in X : x \succ_s y\}$, $\{y \in X : y \succ_s x\}$ are open.*

Axiom 3. *(Limited Indifference) Let $s \in S$. Then $\lambda(\{y \in X : x \sim_s y\}) = 0$ for all $x \in X$.*

Incomplete preferences are an important technical tool of the model, therefore I fix some notation. By incomplete preference I mean any finite, transitive and reflexive binary relation on X. Generic incomplete preference relation is denoted by $\bar{s}$. For a given $\bar{s}$, the set $[\bar{s}] = \{s \in S : \bar{s} \subset s\}$ is the set of extensions of $\bar{s}$ to X. I also denote an incomplete preference relation $\bar{s} = \{(x, y)\}$ by $x \succ y$ and similarly $\bar{s} = \{(x, y), (y, x)\}$ by $x \sim y$, $\bar{s} = \{(x, y), (y, z)\}$ by $x \succ y \succ z$ and so on.[1] I also equip S with topology $\mathcal{T}$, which is the smallest topology, with respect to inclusion, such that $[x \succ y] \in \mathcal{T}$ for all $x, y \in X$.

Knowledge of the consumer is defined by a finite set $K \subset X$. The elements of K are all the alternatives the real preference for which has been revealed to the consumer, meaning that the consumer knows the ranking in which elements of K are with respect to relation s^*. The ranking of the alternatives in K that has been revealed to the consumer provided by the incomplete preference relation $\bar{s^*}_K = s^* \cap (K \times K)$. For simplicity, I assume that $(k_i, k_j) \in \bar{s^*}_K$ and $(k_j, k_i) \in \bar{s^*}_K$ implies that $k_i = k_j$, meaning that the revealed ranking of the alternatives in K is a strict ranking, with no indifference between the elements of K.

Let A be a set of Savage acts, meaning that $A = \{a : S \to [0, 1]\}$ with a standard product topology. I embed X into A as follows: to each alternative $x \in X$ I assign an act a_x defined by $a_x(s) = \lambda(\{y \in X : x \succ y\})$. From now on I write x in place of a_x, so $x(s)$ is the measure of the lower contour set of $x \in X$ with respect to relation s. I assume that the consumer comes equipped with a conditional preference relation $\succeq_K \subset (A \times M)^2$, for all possible K, and define the induced preference relation over acts $\succeq_{K,m}$ as $a \succeq_{K,m} a' \iff (a, m) \succeq_K (a', m)$ and the induced preference over menus $\succeq_{K,x}$ as $m \succeq_{K,x} m' \iff (x, m) \succeq_K (x, m')$.

Definition 1. *Let K be given. Conditional preference relation $\succeq_K$ admits a subjective expected utility representation if and only if there exist scalar $\delta > 0$, a non-atomic probability measure μ_K defined over the Borel sigma field of $[\bar{s^*}_K]$ and a utility function $u : [0, 1] \to \mathbb{R}_+$ such that*

1. *Induced preference over acts $\succeq_{K,m}$ does not depend on m and is represented by an utility function*

$$v_K(x) = \int_{[\bar{s^*}_K]} u(x(s)) d\mu_K(s),$$

[1]Anytime I use any relation symbol, for example $x \succ y$, without any subscript, it denotes an incomplete preference relation defined over this pair of elements.

2. *Induced preference over acts $\succeq_{K,x}$ is represented by an utility function*

$$w_{K,x}(m) = E_K\left[\max_{z\in m}\int_{[\bar{s^*}_K]} u(z(s))d\mu_{K\cup\{x\}}(s)\right],$$

3. *Conditional preference relation $\succeq_{K,x}$ is represented by an utility function*

$$U_K(x,m) = v_K(x) + \delta w_{K,x}(m).$$

Kapera (2024) provides the behavioral axioms under which the conditional preferences of the consumer admit the subjective expected utility representation. I do not repeat these axioms and results, but merely assume that the conditional preferences of the consumer have the subjective expected utility representation. I also assume, that the consumer is bayesian, meaning that the consumer updates their subjective probability according to definition 2.

Definition 2. *The consumer is bayesian if for all finite $K, K' \subset X$ such that $K \subsetneq K'$ and any Borel measurable $C \subset S$*

$$\mu_{K'}(C) = \mu_K(C|\bar{s^*}_{K'}) = \frac{\mu_K(C\cap[\bar{s^*}_{K'}])}{\mu_K([\bar{s^*}_{K'}])}.$$

By proposition 3 of Kapera (2024), if a consumer is bayesian, there exist scalar $\delta > 0$, a probability measure μ defined on Borel sigma field of S and a utility function u such that (δ, μ_K, u) provide a subjective expected utility representation for $\succeq_K$, where $\mu_K(C) = \mu(C|\bar{s^*}_K)$ for all Borel measurable $C \subset S$.

For simplicity, in what follows I always assume that δ is observed and equal to 1. This assumption does not have a qualitative impact on the results, but helps to simplify the notation.

3 Simple observational model

The observational model in this section is the most classical model of the information available to the analyst. Only choice data is observed, much like in the seminal contribution of Afriat (1967), meaning that the set of data, denoted by D, is a finite collection of pairs, each consisting of a menu and the alternative chosen from this menu by the consumer. More formally, data of the consumer is denoted by $D = \{(x_1, m_1), \ldots, (x_n, m_n)\}$, where $x_i \in X$ and $m_i \in M$. The corresponding sequence of the knowledge sets of the consumer $(K_i)_{i=0}^n$ is defined

$K_{i+1} = K_i \cup \{x_{i+1}\}$ and $K_0 = \{x_0\}$ where $x_0 \in X$ is an arbitrary but known alternative, which the consumer knows from the start. The assumption that the consumer knows some alternative x_0 is just a simplification, introduced because the first choice of the consumer is a special case of the behavioral model of the consumer — nothing is learned from the first choice, because there is no other known alternative and therefore the only possible ranking of the known alternatives that can be revealed is that x_0 is simultaneously both the best and the worst known alternative.

The goal of the analyst is to find a subjective expected utility representation (μ, u) that rationalizes D, as defined below.

Definition 3. *Subjective probability measure μ and a utility function u rationalize observed data D if and only if (μ_{K_i}, u) are a subjective expected utility representation of $\succeq_{K_i}$ such that $x_i \succeq_{K_i} z$ for all $z \in m_i$.*

Note, that, even though K is observed, the ranking of elements in K which has been revealed to the consumer, namely $\bar{s^*}_K$ is not observed. Obviously, subjective probability measure μ and utility function u are also unobserved. For simplicity, I assume that the follow up menu is observed, but independent of the choice from the current menu. This assumption does not have an impact on the result in this section.

The following definition 4 and lemma **??** present an important technical tool for the study of rationalizability.

Definition 4. *Let $C \subset S$ and a finite $Y = \{y_1, \ldots, y_m\} \subset X$ be given. Denote Σ_m a set of all permutation of the set $\{1, \ldots, m\}$. Set $C_Y = \{[y_{\sigma(1)} \succ \cdots \succ y_{\sigma(m)}] : \sigma \in \Sigma_m\}$ is a strict partition of C by Y.*

Let $(\bar{K})_{i=0}^{m}$ be a sequence of unique elements of sequence $(K)_{i=0}^{n}$, that is $\bar{K}_0 = K_0$, $\bar{K}_{i+1} = K_j$ where j is a minimum index such that $\bar{K}_i \subsetneq K_j$. For each $\bar{K}_i, \bar{K}_j$ with $i \neq j$ the conditional preference relation of the consumer might be different, therefore the choice data of the consumer is interpreted using the notion of the conditional revealed preference relations, which are defined in definition 5.

Definition 5. *Let $x, y \in X$ and fix some $\bar{K}_j$. Alternative x has been revealed to be conditionally preferred to y under knowledge of $\bar{K}_j$, denoted by $x \succeq^r_{\bar{K}_j} y$, if*

1. *There exists $(x_i, m_i) \in D$ such that $x_i = x$, $y \in m_i$ and $K_i = \bar{K}_j$.*

2. *There exists* $K_{i_1} \subset \bar{K}_j$ *such that* $x, y \in K_{i_1}$ *and* $(x_{i_2}, m_{i_2}) \in D$ *for* $i_2 > i_1$ *such that* $x_{i_2} = x$ *and* $y \in m_{i_2}$.

For a given knowledge of the consumer, conditional revealed preference relation summarizes all of the information which the data reveal regarding the conditional preference for the same knowledge set. By definition 5, there are two types of observations which reveal something regarding the conditional preference relation for a given knowledge set $\bar{K}$. It is either, any observation where x is chosen from menu m and y is not, where the choice has been done with information $\bar{K}$; or any instance where the consumer chooses between two alternatives $x, y \in \bar{K}$ which are already known in both $\bar{K}$ and at the time the choice as been made. Note, that it means that the choice between x, y might have occurred for a strictly smaller or larger information set than $\bar{K}$; as long as both x, y are known both in K and when the choice between them is made, it reveals something about $\succeq_{\bar{K}}$.

The following axiom 4 is an obvious necessary condition for data D to be rationalizable.

Axiom 4. *Conditional revealed preference relation* $\succeq^r_{\bar{K}}$ *satisfies a strong axiom of revealed preference (SARP) if and only if* $\succeq^r_{\bar{K}}$ *can be extended to a transitive, incomplete preference relation on* $\tilde{X} \times \tilde{X}$.

This formulation of a strong axiom of revealed preferences is equivalent to the classical axiom of Houthakker (1950). It is obviously a necessary condition, because when no learning takes place, then preference discovery behaves exactly as a typical preference theory. The interesting question is whether it is also a sufficient condition, because even though the correlations between the beliefs of the consumer can be arbitrary, the consumer is bayesian and as such they correctly anticipate how their beliefs will react to any new information. Therefore the expected impact of any possible future information is already a factor in the current choices of the consumer. The following theorem 1 answers this question.

Theorem 1. *There exists a subjective probability measure* μ *and a utility function* u *rationalizing* D *if and only if* $\succeq_{\bar{K}_i}$ *satisfies SARP for* $i = 1, \ldots, m$.

Proof. Necessity is obvious, I now show sufficiency. I prove it by induction on $|D|$. For $|D| = 1$ theorem is trivially satisfied, therefore assume that it holds for $|D| = n - 1$ and I prove that it holds for $|D| = n$.

Assume towards contradiction, that D is not rationalizable. Since each conditional revealed preference relation satisfies SARP, there exists some incomplete preference relation $\bar{s^*}_{\bar{K}_j}$ such that $\bar{s^*}_{\bar{K}_j}$ rationalizes observed choices between the alternatives from $\bar{K}_j$. By induction assumption $D \setminus (x_n, m_n) = D'$ is rationalizable, so fix some SEU rationalization (μ, u) of D'. If $x_n \in \bar{K}_m$ then then D is rationalizable by SARP, so assume that $x_n \notin \bar{K}_m$. Without loss of generality, assume that each $\bar{s^*}_{\bar{K}_j}$ is known and that $\bar{s^*}_{\bar{K}_j} = k_1 \succ \cdots \succ k_j$.

Let $C = \{C_\sigma : \sigma \in \Sigma\}$ be a strict partition of S by $\bar{K}_m \cup \{x_n\}$, denote $C_j = \{C_\sigma \in C : C_\sigma \cap [\bar{s^*}_{\bar{K}_j}] \neq \emptyset\}$ and for given $k, k' \in \bar{K}_m \cup \{x_n\}$ define

$$C_j^{k \succ k'} = \{C_\sigma \in C_j : k = \sigma^{-1}(i), k' = \sigma^{-1}(j), i < j\}, \quad C_j^{k' \succ k} = C_j \setminus C_j^{k \succ k'}.$$

Now for each $\bar{K}_j$, I can rewrite each revealed relation in terms of this partition, for example

$$k_i \succ_{\bar{K}_j} k_j \iff \sum_{c \in C_j^{k_i \succ k_j}} p_c u_c(k_i) > \sum_{c \in C_j^{k_j \succ k_i}} p_c u_c(k_j),$$

where

$$p_c = \mu(c), \quad u_c(k) = \int_c u(k(s)) d\mu(s).$$

Since D' is rationalizable and D is not, there must exist $k_j \in \bar{K}_m$ such that $x_n \succ_{\bar{K}_n} k_j$ is revealed but

$$\sum_{c \in C_m^{x_n \succ k_j}} p_c u_c(x_n) < \sum_{c \in C_m^{k_j \succ x_n}} p_c u_c(k_j),$$

is not satisfied. Let $\epsilon \in (0, 1)$ satisfy

$$\sum_{c \in C_m^{x_n \succ k_j}} p_c u_c(x_n) > \epsilon \sum_{c \in C_m^{k_j \succ x_n}} p_c u_c(k_j),$$

and denote $k = \bar{K}_m \setminus \bar{K}_{m-1}$. Define for each $c \in C_m^{k_j \succ x_n}$ the set of all elements of C that differ form c only by placement of k, meaning $C_c = \{c' \in C_{m-1} : \forall_{k_i, k_l \neq k} c' \subset [k_i \succ k_l] \iff c \subset [k_i \succ k_l]\}$. Now it is sufficient to define μ_0 on partition C as follows. For all $c_\sigma \in C$ such that there does not exist $c \in C_m^{k_j \succ x_n}$ such that $c_\sigma \in C_c$, define $\mu_0(c_\sigma) = \mu(c_\sigma)$. For all $c \in C_m^{k_j \succ x_n}$ define $\mu_0(c) = \epsilon \mu(c)$. Now for each $c \in C_m^{k_j \succ x_n}$ and for all $c \neq c' \in C_c$ define

$$\mu_0(c') = \frac{\sum_{c'' \in C_c} \mu(c'')}{\sum_{c'' \in C_c} \mu(c'') - (1 - \epsilon)\mu(c)} \mu(c').$$

By theorem 1 of Kapera (2022) there exists subjective probability measure μ' extending μ_0 and preserving each $u_c(k_i)$. Therefore (μ', u) rationalize D and the proof is finished.

□

By theorem 1 the strong axiom of revealed preferences is both a necessary and sufficient condition for D to be rationalizable. As such it is the only testable restriction of the model, and even then it does not apply to the whole of the data, but merely applied to each conditional revealed preference, which is a strictly weaker condition than SARP for the whole data D at once. Moreover, this is not a testable condition of the preference discovery model itself, because there is no condition on the observed behavior between different information sets, other than consistency of choices between the alternatives whose relative ranking has already been discovered and does not change with new information.

4 Extended model

In the simple observational model presented in section 3, the bayesian anticipation of the changes in their own beliefs by the consumer fails to impose any meaningful, testable conditions on the observed behavior of the consumer between different knowledge sets. There are two reasons for this failure. Firstly, the usual limited observability problem, meaning that every time any ex ante unknown alternative is chosen by the consumer, their knowledge set is updated. As such, data can only impose just a single condition on the current beliefs of the consumer, which is not enough. Secondly, there is not enough structure on the set X to identify how learning changes the beliefs of the consumer. At best, we can reveal the order of the alternatives in K, but just their order.

To solve these two problems, in this section I extend the simple observational model. Firstly, I assume that we not only observe the follow up menu, but that the follow up menu depends on the choice from the current menu and can be chosen by the consumer. Secondly, I extend the set of possible consumption alternatives to $X \cup \mathbb{R}_+$, where the real numbers are possible monetary payoffs. I assume that the consumer knows their preference for monetary payoffs, however this knowledge has no impact on their preference for elements of X. Finally, I allow the analyst to observe hypothetical choices of the consumer. A choice is hypothetical, if there is no learning from this choice, and the chosen follow up menu is not realised.

Formally, let $\tilde{X} = X \cup [0,1]$ with metric $\tilde{d}$ defined as $\tilde{d}(x,y) = d(x,y)$ for $x, y \in X$, $\tilde{d}(x,y) = d_e(x,y) = |x-y|$ for $x, y \in [0,1]$ and $\tilde{d}(x,y) = \infty$ for $x \in X$ and $y \in [0,1]$ or vice versa. I extend conditional preferences of the consumer so that $x \in [0,1]$ is indifferent to a constant act $x \in A$. Set of possible follow up menus M now consists of finite subsets of $\tilde{X}$. I also define set of possible current menus, $\tilde{M}$, as a collection of finite subsets of $\tilde{X} \times M$. Data of the consumer $D = \{((x_1, m_1), \tilde{m_1}, K_1), \ldots, ((x_n, m_n), \tilde{m_n}, K_n)\}$, where $(x_i, m_i) \in \tilde{X} \times M$, $\tilde{m_i} \in \tilde{M}$ and K_i is a finite subset of X satisfying $K_{i+1} \subset K_i \cup \{x_i\}$. Again I assume $K_0 = \{x_0\}$ for some arbitrary $x_0 \in X$.

Even outside of the introduction of monetary payoffs[2], there are two significant changes in the definition of the data. Firstly, the consumer now chooses a pair of a current consumption alternative x_i together with a follow up menu m_i, and the objects in the current menu $\tilde{m_i}$ are such pairs. However, a follow up menu $m_i \subset \tilde{X}$ so it only consists of consumption alternatives, meaning that the consumer only looks one period ahead in their consumption. It can be interpreted as the consumer being bayesian but not wholly sophisticated, or that the consumer simply does not have an information regarding the possible future menus other than this one. Note, that I do not demand that the follow up menu that the consumer chooses has any relation to the actual menu in the next period. It might be assumed that the actual next period menu consists of the same consumption alternatives as the follow up menu that the consumer has chosen, but I do not make this assumption as it is not necessary for my results.

Secondly, note that K_{i+1} is now only a subset of $K_i \cup \{x_i\}$, meaning that either $K_{i+1} = K_i \cup \{x_i\}$ or $K_{i+1} = K_i$. In the second case, the choice in period i is a hypothetical choice. The consumer makes a choice, but it is never actually realized, so no learning takes place. We might interpret this situation as placing an online order, which by mistake never arrives to the consumer. No matter the interpretation, a hypothetical choice means that some choice has been made, but the consumer does not learn from this choice no matter what has been chosen.

[2]It does not have to be $[0,1]$. Any compact and connected subset of the real line is enough, but this exact choice makes everything easier because it coincides with the set of possible consequences in the formulation of A.

All of the definitions that has not been explicitly altered stay just as in section 3. The definitions of rationalization of the data and of the conditional revealed preference relation admit only slight changes compared to definitions 3 and 5 in the previous section, stated below.

Definition 6. *Subjective probability measure μ and a utility function u rationalize observed data D if and only if (μ_{K_i}, u) are a subjective expected utility representation of $\succeq_{K_i}$ such that $(x_i, m_i) \succeq_{K_i} (z, m_z)$ for all $(z, m_z) \in \tilde{m}_i$.*

Definition 7. *Let $(x, m_x), (y, m_y) \in \tilde{X} \times M$ and fix some $\bar{K}_j$. Pair (x, m_x) has been revealed to be conditionally preferred to (y, m_y) under knowledge of $\bar{K}_j$, denoted by $(x, m_x) \succeq^r_{\bar{K}_j} (y, m_y)$, if*

1. *There exists $((x_i, m_i), \tilde{m}_i, K_i) \in D$ such that $(x_i, m_i) = (x, m_x)$, $(y, m_y) \in \tilde{m}_i$ and $K_i = \bar{K}_j$.*

2. *There exists $K_{i_1} \subset \bar{K}_j$ such that $x, y \in K_{i_1} \cup [0, 1]$ and $((x_{i_2}, m_{i_2}), \tilde{m}_{i_2}, K_{i_2}) \in D$ for $i_2 > i_1$ such that $(x_{i_2}, m_{i_2}) = (x, m_x)$ and $(y, m_y) \in \tilde{m}_{i_2}$.*

3. *Both $x, y \in [0, 1]$, $x > y$ and $m_x = m_y$.*

Compared to definitions in section 3, the only difference in definition 6 is that it has been adapted to the new structure of data D. Definition 7 is not only adapted, but also extended by adding the known relation between all the monetary payoffs. Note, that although other then that the definition is the same, the possibility of hypothetical choices imply that each $\succeq^r_{\bar{K}_j}$ might consist of significantly more observations, and in turn theorem 2 shows, that in the extended model there are restrictions that are testable across periods.

Theorem 2. *Let arbitrary rationalizable data D be given and be such that $\succeq_{\bar{K}_i}$ satisfies SARP for $i = 1, ..., m$. There exist a finite $\tilde{D}$ such that $D \subset \tilde{D}$, $\tilde{D} \setminus D$ consists only of hypothetical choices and each $\succeq_{\bar{K}_i}$ satisfies SARP, but $\tilde{D}$ is not rationalizable.*

Proof. Assume that $D = \{((x_1, m), \tilde{m}_1, K_1), ((x_2, m), \tilde{m}_2, K_2)\}$, where $m = \{x_0, x_1, x_2\}$, $(x_2, m) \in \tilde{m}_1$, $(x_1, m) \in \tilde{m}_2$ and $K_1 = \{x_0\}$, $K_2 = \{x_0, x_1\}$. Define the following hypothetical choices

$$c_1 = ((x_1, \{\beta\}), \{(x_1, \{\beta\}), (x_1, \{x_2\}), K_1\})$$

and

$$c_2 = ((x_1, \{x_1\}), \{(x_1, \{x_1\}), (x_1, \{\beta\})\}, K_2).$$

Now let $\tilde{D} = \{c_1, ((x_1, m), \tilde{m}_1, K_1), c_2, ((x_2, m), \tilde{m}_2, K_2)\}$ and assume that U is the SEU representation of $\succeq_{K_2}$ and $\bar{U}$ is the SEU representation of $\succeq_{K_2,m}$. Then by construction

$$U(x_1, m) > 2\bar{U}(\beta, m) > U(x_2, m),$$

therefore $\tilde{D}$ is not rationalizable. It is an easy exercise to generalize this proof to any D by induction.

$\square$

Theorem 2 does not only state, that in the extended model has some testable across periods restrictions. It is significantly stronger, and states that a finite number of hypothetical choices can always falsify any observed data.

5 Identification

The extended observational model presented in section 4 is powerful enough to allow not only to impose restrictions that are testable between different knowledge sets, but also to identify the beliefs of the consumer, their remaining taste uncertainty and most significantly, their conditional beliefs. This last property of the extended model is the most significant, since it allows to identify not only current or past, but also future behavior of the consumer. It is also this property that is responsible for the existence of the restrictions testable across different knowledge sets.

The main idea behind the identification presented in this section follows Cooke (2016), who notices that the observed experimentation and choices with option menus, which in the language of section 4 would be the menus of form $\{x, \beta\}$ where $x \in X$ and $\beta \in [0, 1]$, allow us to identify the remaining taste uncertainty and parameters of the model.

There are multiple differences in the formulation of the subjective expected utility model of a taste uncertain consumer between the model presented in section 2 and the model of Cooke (2016). These differences are described in Kapera (2024), but the main important difference is that in the Cooke (2016) model, the consumer learns from consumption of x its cardinal utility, meaning that all of the uncertainty with respect to x has been resolved, and Kapera (2024) assumes ordinal learning from consumption, which resolves only the uncertainty regarding the relative ranking of x with respect to other known alternatives, but not the unknown ones. Nevertheless with slight modifications, I am able to obtain similar identification results.

Definition 8. *A menu $m \in M$ is a valuation menu if $m = \{\beta\}$ for $\beta \in [0,1]$.*

Definition 9. *A menu $m \in M$ is an option menu if $m = \{x, \beta\}$ for $x \in X$ and $\beta \in [0,1]$.*

Valuation and option menus together allow for a full identification of both conditional and unconditional beliefs of the consumer. In a way of example, assume that $x_1 \in K$, $x_2 \notin K$, and denote $EU(x) = \int_S u(x(s))d\mu_K$, $EU(x|y_1 \succ y_2) = \int_{S\cap[y_1 \succ y_2]} u(x(s))d\mu_K$, meaning that $EU(x)$ is the unconditional expected utility of x and $EU(x|y_1 \succ y_2)$ is the expected utility of x conditionally on learning that $y_1 \succ y_2$. Using valuation menus, we can easily identify unconditional expected utility of x_1, x_2 as

$$(x_1, \{x_1\}) \sim_K (x_1, \{\beta_1\}), \quad (x_2, \{x_2\}) \sim_K (x_2, \{\beta_2\}),$$

so that $EU(x_1) = \beta_1$ and $EU(x_2) = \beta_2$. Similarly, for conditional beliefs

$$(x_2, \{x_1, x_2\}) \sim_K (x_2, \{\beta_1^*, x_2\}), \quad (x_2, \{x_1, x_2\}) \sim_K (x_2, \{x_1, \beta_2^*\}),$$

identifies $EU(x_1|x_1 \succ x_2) = B_1^*$ and $EU(x_2|x_2 \succ x_1) = \beta_2^*$. It is slightly less obvious how to identify the expected utility of x_i conditionally on learning that it is the worse out of x_1, x_2. In order to identify it, I first need to obtain the subjective probability of $x_1 \succ x_2$ as

$$(x_2, \{x_2, \beta_2^*\}) \sim_K (x_2, \{\tilde{\beta}\}) \implies \tilde{\beta} = \mu([x_1 \succ x_2])\beta_2 + \mu([x_2 \succ x_1])\beta_2^*,$$

so that $\mu([x_1 \succ x_2]) = \frac{\beta_2^* - \tilde{\beta}}{\beta_2^* - \beta_2}$ and from the equation that

$$EU(x_1) = \mu([x_1 \succ x_2])EU(x_1|x_1 \succ x_2) + (1 - \mu([x_1 \succ x_2]))EU(x_1|x_2 \succ x_1),$$

I can identify $EU(x_1|x_2 \succ x_1)$ as

$$EU(x_1|x_2 \succ x_1) = \frac{\beta_1 - \beta_1^* \left(\frac{\beta_2^* - \tilde{\beta}}{\beta_2^* - \beta_2}\right)}{1 - \frac{\beta_2^* - \tilde{\beta}}{\beta_2^* - \beta_2}}.$$

Using valuation and option menus, I am also able to identify the remaining taste uncertainty of the consumer.

Definition 10. *Let K be given. The remaining taste uncertainty of the consumer is $\mu([\bar{s^*}_K])$.*

Definition 10 of remaining taste uncertainty is natural. If the consumer fully discovered their preferences s^*, then the remaining taste uncertainty by this definition is 0. With no taste information whatsoever, it is 1, and for some partial knowledge K, value of $1 - \mu([\bar{s^*}_K])$ is the measure of all of the preferences that has been excluded from potentially being the real preferences of the consumer by experience of K.

We can obviously identify $\bar{s^*}_{\bar{K}_j}$ for each $j = 1, \ldots, m$ by presenting the consumer with a sequence of binary choices from $\bar{K}_j$. Let $k = \bar{K}_{j+1} \setminus \bar{K}_j$. As shown above, we can identify the subjective probability $\mu_{\bar{K}_j}([k \succ k_j])$ for each $k_j \in \bar{K}_j$, therefore we can calculate $\mu_{\bar{K}_j}([\bar{s^*}_{\bar{K}_j}])$. Now it suffices to note, that

$$\mu([\bar{s^*}_{\bar{K}_m}]) = \prod_{i=1}^{m} \mu_{\bar{K}_{i-1}}([\bar{s^*}_{\bar{K}_i}]),$$

to show that the remaining taste uncertainty of the consumer is also identifiable by valuation and option menus.

References

Afriat, S. (1967). The Construction of a Utility Function from Demand Data. International Economic Review, 8, 67.

Braga, J. and Starmer, C. (2005). Preference anomalies, preference elicitation and the discovered preference hypothesis. Environmental and Resource Economics, 32, 55-89.

Braga, J., Humphrey, S. J. and Starmer, C. (2009). Market experience eliminates some anomalies—and creates new ones. European Economic Review, 53(4), 401-416.

Bruni, L. and Sugden, R. (2007). The road not taken: how psychology was removed from economics, and how it might be brought back. The Economic Journal, 117(516), 146-173.

Butler, D. J. and Loomes, G. C. (2007). Imprecision as an account of the preference reversal phenomenon. American Economic Review, 97(1), 277-297.

Cooke, K. (2017). Preference discovery and experimentation. Theoretical Economics, 12(3), 1307-1348.

Cox, J. C. and Grether, D. M. (1996). The preference reversal phenomenon: Response mode, markets and incentives. Economic Theory, 7(3), 381-405.

Czajkowski, M., Hanley, N., and LaRiviere, J. (2015). The effects of experience on preferences: theory and empirics for environmental public goods. American Journal of Agricultural Economics, 97(1), 333-351.

Frick, M., Iijima, R., and Strzalecki, T. (2019). Dynamic random utility. Econometrica, 87(6), 1941-2002.

Gul, F., and Pesendorfer, W. (2006). Random expected utility. Econometrica, 74(1), 121-146.

Houthakker, H. S. (1950). Revealed preference and the utility function. Economica, 17(66), 159-174.

Humphrey, S. J., Lindsay, L. and Starmer, C. (2017). Consumption experience, choice experience and the endowment effect. Journal of the Economic Science Association, 3, 109-120.

Kahneman, D. and A. Tversky. (1979). Prospect Theory: An Analysis of Decision under Risk. Econometrica, 47(2), 263-292

Kapera, M. (2024). Learning, experimentation and the convergence of the discovered preferences. INE PAN Working Paper no. 56.

Kapera, M. (2022). Learning own preferences through consumption. No 2022-074, KAE Working Papers, Warsaw School of Economics, Collegium of Economic Analysis.

Kreps, D. M. (1979). A representation theorem for "preference for flexibility". Econometrica: Journal of the Econometric Society, 565-577.

Lichtenstein, S. and Slovic, P. (2006). The construction of preference: An overview. Cambridge University Press.

Loomes, G., S. Orr, and R. Sugden. (2009). Taste uncertainty and status quo effects in consumer choice. Journal of Risk and Uncertainty, 39, 113–135

Kingsley, D.C., and T.C. Brown (2010). Preference uncertainty, preference learning, and paired comparison experiments. Land Economics 86(3), 530–544.

Plott, Charles R. (1996). "Rational individual behaviour in markets and social choice processes: the discovered preference hypothesis". In: The rational foundations of economic behaviour: Proceedings of the IEA Conference held in Turin, Italy. Ed. by Kenneth J. Arrow. Vol. 114. New York: St. Martin's Press; London: Macmillan Press in association with the International Economic Association, 225–250

Kreps, D. M., and Porteus, E. L. (1978). Temporal resolution of uncertainty and dynamic choice theory. Econometrica: journal of the Econometric Society, 185-200.

Kuilen, Gijs van de (2009). Subjective probability weighting and the discovered preference hypothesis. Theory and Decision 67, 1–22