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Abstract

In this article I study whether the interim preferences of the consumer can be expected to converge to their real preferences in the process of preference discovery. I construct a subjective expected utility model of the consumer, where the uncertainty results from the imperfect knowledge of their own preferences. This uncertainty is partially resolved by experimental consumption. Under the assumption that the subjective probability of the consumer satisfies learning monotonicity, I identify the equivalent conditions for the consumer to experiment. My results show that the interim preferences never fully converge to the real preferences of the consumer. Instead, the preference discovery either terminates, meaning that the consumer ceases to experiment, or only experiments within some neighbourhood of the best currently known alternative, and never sufficiently explores their preferences.

1 Introduction

Rationality of the consumer is one of the key assumptions in the economic theory. It states that the choices of the consumer are an outcome of some optimizing behaviour. This is what Simon (1976) calls substantive rationality, in contrast

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to procedural rationality which is typically assumed in psychology. Procedural rationality is significantly less demanding, as it treats choice of the consumer as rational if only it is an outcome of some appropriate mental deliberation. One of the main reasons for this discrepancy are the observed paradoxes of choice, most notably preference reversal, which in words of Grether and Plott (1979) is a paradox that seem to contradict the existence of any optimizing behaviour, and as a result the existence of the consumer preferences (see Lichtenstein and Slovic 2006 for a comprehensive review).

One justification for the economic notion of rationality is provided by the preference discovery hypothesis of Plott (1996). It states, that the consumer has some well defined and stable real preference relation which represent their choices. However, this relation is ex ante unknown to the consumer and only discovered by consumption experience. Plott (1996) argues that in experiments the subjects are asked to choose between alternatives which they never experience in everyday lives and as such they make mistakes. However, as first shown by Cox and Grether (1996) those mistakes are less frequent in the setting with repeated choices and incentives to learn. Under normal market conditions both of those conditions are satisfied, meaning that the consumer has to make repeated choices between the same alternatives and also has an incentive to learn, because better knowledge of their own preferences leads to an increased utility from future choices. Therefore, Plott (1996) concludes that in everyday choices the consumers know their own preferences and the assumption of the substantive rationality of the consumer is justified.

Preference discovery hypothesis is generally supported by empirical studies. There is a lot of literature on this topic, for example Butler (2007), Czajkowski et al. (2015) and Humphrey et al. (2017) which shows that not only consumer choices stabilize in repeated experiments, but also a lot of known paradoxes of choice, including preference reversal and WTA/WTP disparity, are less persistent with each repetition. However, even if we accept that preference discovery hypothesis is true, it is not at all clear whether it is successful as a defence of substantive rationality of the consumer in market conditions.

Obviously, there are many situations in which the consumer is unlikely to know their own preference, like introduction of a new product to the market or situations when experimentation is costly (for example, preference for romantic partners).

For this reason Rizzo and Whitman (2018) consider substantive rationality from the perspective of preference discovery as a process, not a state. More significant is the objection voiced by Braga and Starmer (2005), Bruni and Sugden (2007) and Braga et al. (2009) that even if preference discovery takes place, we have no idea whether the process of preference discovery converges to the real preferences to the consumer. It is very much possible that the consumer at some point ceases to experiment without exploring all of the alternatives, because the incentive to learn provided by the market might not be strong enough for the consumer to explore the whole range of possibilities. This hypothesis is supported by results of Delaney et al. (2020), who show that irrespective to time horizon of the study, the fraction of the alternatives which the consumer tried out stabilizes around 87%.

The purpose of this article is to answer the question, whether there exist any reasonable theoretical conditions regarding the learning behaviour of the consumer, under which the consumer fully discovers their own preferences. I only consider this question under what Plott (1996) considers as market conditions, meaning that the only incentive for the consumer to learn comes from the expectation of a higher utility from future choices. As such, I do not consider the possibility that the consumer exhibits a preference for experimentation itself. To the best of my knowledge, the only other contribution to study this questions is Delaney et al. (2014). However, the authors only considers the possibility of preference discovery depending in the properties of the sequence of menus from which the consumer chooses, not how the consumer learns and updates their probabilistic beliefs regarding their own taste, and under very restrictive assumptions.

In order to answer this question, I obtain a subjective expected utility representation of the conditional preferences, which are the preferences of the consumer under partial information regarding their taste. This approach is in line with existing studies of a taste uncertain consumer, starting with the seminal contributions of Kreps (1979) and Dekel et al. (2001), with the more recent contributions by Piermont et al. (2016) and Cooke (2017). The last two contributions are especially relevant, since both Piermont et al. (2016) and Cooke (2017) condition the resolution of the taste uncertainty on consumption. I largely follow Cooke (2017) in my modelling assumptions, and similarly to him I consider the consumer as choosing a first period consumption together with a follow up menu, where the choice from the follow up menu makes use of the learning from the consumption in the first period.

I differ with Cooke (2017) in several points. Firstly, I assume that the consumer learns not the cardinal utility from the consumed alternatives, but merely the ordinal ranking of the alternative with respect to other known alternatives. This is an important difference, since due to this assumption, the consumption in my model does not fully resolve the taste uncertainty with respect to the already consumed alternatives, as the ranking of this alternative with respect to the unconsumed alternatives remains unknown. Secondly, I do not assume lotteries, instead I treat each alternative as a Savage (1954) act, that assigns to each possible preference relation the position of the chosen alternative in the ranking of all the alternatives by this relation. By extension, even though the representation itself is similar to the one Cooke (2017) obtains, both the utility function and the subjective probability measure are very different objects. One significant difference to Savage (1954) is that I consider the subjective probability to be countably additive. In this respect, my model is in line with the construction of Kapera (2022).

After obtaining the representation, I consider two questions. Firstly, I am interested in the identification of the experimental behaviour of the consumer, meaning what are the conditions, under which the consumer represented by the model prefers some unknown alternative, instead of the alternative for which their preference has already been partially resolved. Secondly, I consider the main question of this article, that is whether there exist any reasonable assumptions regarding the learning behaviour of the consumer, under which the consumer could be expected to fully discover their preferences.

I answer both these questions using an additional assumption of learning monotonicity, which is a very natural restriction on the possible correlations of the beliefs of the consumer. Under this restriction, I identify an equivalent condition for the consumer to choose to experiment, which is that the consumer needs to believe that their preference for the alternative chosen to experiment is on average sufficiently highly correlated with their preference for other alternatives. Finally, I show that under learning monotonicity, the consumer never fully resolves their taste uncertainty. Depending on the weight that the consumer assigns to the current consumption relative to the future one, it is possible under learning monotonicity that the consumer never ceases to experiment, but they do not explore the full range of the alternatives. As a result, the consumer might only be expected to discover their preference for some subset of the alternatives.

The structure of this article is as follows. I begin with a very short introduction to the model, which contains all the elementary definitions that are necessary. I next turn to the representation of the conditional preferences of the consumer in section 3. Finally, in section 4 this representation is applied to the identification of the experimental behaviour of the consumer, and to the question of the convergence of the conditional preferences to the real preferences of the consumer.

2 Elementary definitions

The objects of choice in the model are represented by set X. I assume X comes equipped with metric d and that it is compact and connected in the topology induced by d. I also assume there exists a non-atomic measure λ defined on the sigma field of Borel subsets of X, such that $\lambda(A) > 0$ for every open subset $A \subset X$. Since X is compact, without loss of generality I can assume that $\lambda(X) = 1$.

Set of possible preferences is denoted by S, and a generic preference relation by $s \in S$. Possible preferences are all the binary relations $s \subset X \times X$ that satisfy axioms 1–3 defined below. Elements of S are all the preference relations that might be the real preferences of the consumer, meaning that the real preference relation of the consumer, denoted s^*, is an element of S, but it is ex-ante unknown to the consumer which element of S it is. I denote weak preference, strict preference, and indifference with respect to a given $s \in S$ by respectively $\succeq_s$, $\succ_s$ and $\sim_s$.

Axiom 1. *(Rationality) Let $s \in S$. Then s is complete, reflexive and transitive.*

Axiom 2. *(Continuity) Let $s \in S$. For each $x \in X$ sets $\{y \in X : x \succ_s y\}$, $\{y \in X : y \succ_s x\}$ are open.*

Axiom 3. *(Limited Indifference) Let $s \in S$. Then $\lambda(\{y \in X : x \sim_s y\}) = 0$ for all $x \in X$.*

The consumer partially discovers their real preferences through consumption. Let $K = \{k_1, \ldots, k_n\} \subset X$ be a set of the alternatives that are already known to the consumer. Unless specified otherwise, I always assume that $2 \leq |K| < \infty$. From the consumption of the alternatives in K the consumer learns the ordinal preference ranking of those alternatives. Formally, the knowledge of the consumer is represented by an incomplete preference relation $\bar{s^*}_K = s^* \cap K \times K$, that is the restriction of their real preference relation s^* to the subset of known alternatives K.

By incomplete preference relation I consider any binary relation on X that is finite and transitive. I denote a generic incomplete preference relation by $\bar{s}$. For any set $Y = \{y_1, \dots, y_n\} \subset X$ and relation $s \in S$ I denote by $\bar{s}_Y$ a restriction of s to Y, meaning that $\bar{s}_Y$ is an incomplete preference relation $\bar{s}_Y = s \cap (Y \times Y)$. Conversely, for a given incomplete preference relation $\bar{s}$ I denote by $[\bar{s}] = \{s \in S : \bar{s} \subset s\}$ the set of all the extensions of $\bar{s}$ to X and for a given set $Y \subset X$ the set of extensions of $\bar{s}$ to Y is given by $[\bar{s}|Y]$. For any set $Y = \{y_1, y_2\}$ with only two elements and s such that $(y_1, y_2) \in s, (y_2, y_1) \notin s$ (respectively $(y_1, y_2) \notin s, (y_2, y_1) \in s$ and $(y_1, y_2) \in s, (y_2, y_1) \in s$) I denote $\bar{s}_Y$ by $y_1 \succ y_2$ (respectively $y_2 \succ y_1$ and $y_1 \sim y_2$). Similarly I sometimes use $x \succ y \succ z$ to denote the smallest (with respect to inclusion) incomplete preference relation $\bar{s}$ such that $\{(x, y), (y, z)\} \subset \bar{s}$.

Finally, I equip S with a topology $\mathcal{T}$ generated by the family of all the extensions of relations, meaning that $\mathcal{T}$ is the smallest topology such that for all $x, y \in X$ set $[x \succ y] \in \mathcal{T}$.

3 Conditional preferences

Let A be a set of Savage acts, meaning that $A = \{a : S \to [0, 1]\}$ with a standard product topology. I embed X into A as follows: to each alternative $x \in X$ I assign an act a_x defined by $a_x(s) = \lambda(\{y \in X : x \succ y\})$. From now on I write x in place of a_x, so $x(s)$ is the measure of the lower contour set of $x \in X$ with respect to relation s. Act $a \in A$ is a simple act if its image $a(S)$ is finite and for each $p \in a(S)$ there exists an incomplete preference relation $\bar{s}_p$ such that $a^{-1}(p) = [\bar{s}_p]$. In the special case where the image of $a \in A$ is a singleton, it is a constant act. Abusing notation a little, for any $p \in [0, 1]$, the constant act $a \in A$ such that $a(S) = \{p\}$ is also denoted by p. I also define for any two acts $a, b \in A$ and set

$C \subset S$ the mixture of a, b over C as

$$a_C b(s) = \begin{cases} a(s), & s \in C \\ b(s), & s \notin C. \end{cases}$$

Moreover let M be a collection of all compact subsets of X and equip it with a Hausdorff topology.[1] Each element $m \in M$ is interpreted as a menu. Notice, that menu are subsets of X, not A. It is the case, because only the alternatives in X are the ones that the consumer can choose, and their preference for which they discover. Acts $a \in A \setminus X$ are hypothetical options only.

I assume that the consumer comes equipped with a conditional preference relation defined over the space of acts and menu pairs, that is $\succeq_{\bar{s}} \subset (A \times M)^2$, for each possible incomplete preference relation $\bar{s}$. I consider the space $A \times M$ with a standard product topology. For the special case of $\bar{s^*}_K$ I use $\succeq_K$ instead of $\succeq_{\bar{s^*}_K}$. Of course, only $\succeq_K$ can be observed, it is the relation conditional on the realised knowledge of the consumer. For $\bar{s} \neq \bar{s^*}_K$ the conditional relation $\succeq_{\bar{s}}$ is a hypothetical preference, and reflects what the conditional preference relation of the consumer would be if the consumer has learned $\bar{s}$ instead of $\bar{s^*}_K$. It is of purely technical importance.

The following axiom 4 is standard and by theorem of Debreu (1954) it guarantees that there exists a continuous utility function representing $\succeq_{\bar{s}}$.

Axiom 4. *$\succeq_{\bar{s}}$ is complete, transitive, reflexive and continuous.*

I define for each $\bar{s}$, $m \in M$ and $a \in A$ two induced preference relations, that is the induced preference relation over acts $\succeq_{\bar{s},m}$, defined as $a \succeq_{\bar{s},m} a' \iff (a, m) \succeq_{\bar{s}} (a', m)$ and the induced preference over menus $\succeq_{\bar{s},a}$ defined as $m \succeq_{\bar{s},a} m' \iff (a, m) \succeq_{\bar{s}} (a, m')$.

Axiom 5. *Let $x \succeq_{\bar{s},m} y$. Then $x \succeq_{\bar{s},m'} y$ for all $m' \in M$.*

By axiom 5 relation $\succeq_{\bar{s},m}$ does not depend on the choice of $m \in M$, so I write $\succeq_{\bar{s},M}$ instead of $\succeq_{\bar{s},m}$. This axiom reflects the fact, that if the consumer only

[1]Hausdorff topology is a topology induced by the Hausdorff metric, that is

$$d_H(m, m') = \max \left\{ \sup_{a \in m} \inf_{a' \in m'} d(a, a'), \sup_{a' \in m'} \inf_{a \in m} d(a, a') \right\}.$$

evaluates a first period consumption ignoring the follow up menu, their choice does not depend on the menu. The following axioms 6–10 are the necessary axioms to obtain the subjective expected utility representation of the induced preference over acts.

Axiom 6. *Let $a, a', b, b' \in A$ and $C \subset S$ be open. Then* $a_C b \succeq_{\bar{s},M} a'_C b \iff a_C b' \succeq_{\bar{s},M} a'_C b'$.

Axiom 7. *Let $p, q \in [0, 1]$. Then $p > q$ if and only if $p_C a \succ_{\bar{s},M} q_C a$ for all $a \in A$ and open $C \subset S$.*

Axiom 8. *Let $p > q$, $p' > q'$ for $p, q, p', q' \in [0, 1]$ and $C, C' \subset S$ be open. Then* $p_C q \succ_{\bar{s},M} p_{C'} q \iff p'_C q' \succ_{\bar{s},M} p'_{C'} q'$.

Axiom 9. *For all $p, q, r \in [0, 1]$, $a \in A$ and open $C \subset S$*

$$(q_C a \prec_{\bar{s},M} r \prec_{\bar{s},M} p_C a) \implies (\exists_{C' \subset S} \colon r \sim_{\bar{s},M} p_{C'} q_C a),$$

for some open C' disjoint from C.

Axiom 10. *Let $[\bar{s}'] = C \subset S$. Then $a_C b \sim_{\bar{s},M} a_C b'$ for all $a, b, b' \in A$ if and only if $C \cap [\bar{s}] = \emptyset$.*

Axioms 6, 7 and 8 are adapted versions of Savage (1954) axioms. Axiom 9 is Abdellaoui and Wakker (2020) solvability axiom in their modified Savage (1954) model. With my definition of the consequences of acts, the remaining Savage (1954) axioms are already implied by the axioms that I do assume. Axiom 10 states that any event excluded by what is already known to the consumer is null. I am now able to state representation theorem for the induced preference relation over acts.

Proposition 1. *Relation $\succeq_{\bar{s}}$ satisfy axioms 4-10 if and only if there exists a probabilistic measure $\mu_{\bar{s}}$ on a sigma field of Borel subsets of $[\bar{s}]$ and a continuous, strictly increasing utility function $u : [0, 1] \to \mathbb{R}_+$ such that for all Borel measurable a, a'*

$$a \succeq_{\bar{s},M} a' \iff \int_{[\bar{s}]} u(a(s)) d\mu_{\bar{s}}(s) \geq \int_{[\bar{s}]} u(a'(s)) d\mu_{\bar{s}}(s).$$

Moreover, all simple acts and all acts associated with $x \in X$ are measurable.

Proof. Let $\bar{A} \subset A$ be a set of all simple acts. By axiom 9 for each $a \in \bar{A}$ there exists $p_a \in [0,1]$ such that $p_a \sim_{\bar{s},M} a$. Let $p : \bar{A} \to [0,1]$ be a function defined as $p(a) = p_a$ and define

$$\tilde{\mu}_{\bar{s}}([\bar{s}']) = p(1_{[\bar{s}']}0).$$

Note, that by axiom 8 p is a utility function representing $\succeq_{\bar{s},M}$ on $\bar{A}$. By lemma 11 of Abdellaoui and Wakker (2020), axioms 4–9 imply that this $\tilde{\mu}_{\bar{s}}$ is additive. Therefore

$$p(1_{[\bar{s}']}0) + p(0_{[\bar{s}']}1) = 1.$$

Therefore $\tilde{\mu}_{\bar{s}}$ satisfies the assumptions of theorem 1 of Kapera (2022) and by this theorem there exists a unique extension of $\tilde{\mu}_{\bar{s}}$ to a probabilistic measure $\mu_{\bar{s}}$ defined on the Borel sigma field of S such that $\mu_{\bar{s}}([\bar{s}']) = \tilde{\mu}_{\bar{s}}([\bar{s}'])$. By axiom 10 this measure is equal to 0 for any subset of S disjoint with $[\bar{s}]$. Moreover, from lemma 13 of Abdellaoui and Wakker (2020), axioms 4–9 imply that there exists a continuous utility function $u : [0,1] \to \mathbb{R}_+$ such that

$$(*) \quad p(q_{[\bar{s}']}0) = u(q)p(1_{[\bar{s}']}0).$$

Now define $\bar{U} : \bar{A} \to [0,1]$ as

$$\bar{U}(a) = \sum_{x \in a(S)} x \mu_{\bar{s}}(a^{-1}(x)).$$

By additivity of p and $(*)$, $\bar{U}$ represents $\succeq_{\bar{s},M}$ on $\bar{A}$. Now fix arbitrary $x \in X$ and a sequence $(B_i)_{i=2}^{\infty}$ such that $B_i \subset X$, $|B_i| = i$, $B_i \subset B_{i+1}$ and $B = \bigcup_{i=2}^{\infty} B_i$ is dense in X. For each B_i let

$$D_i = \{[\bar{s}'] : \exists_{s \in S} \ \bar{s}_{B_i} = \bar{s}'\},$$

and denote $D_i(s) = D$ where $D \in D_i$ satisfies $s \in D$. Define two sequences of simple acts $\bar{a}_i$ and $\underline{a}_i$ as

$$\bar{a}_i(s) = \max_{s' \in D_i(s)} x(s'), \quad \underline{a}_i(s) = \min_{s' \in D_i(s)} x(s').$$

Both sequences converge pointwise to x and are measurable with respect to $\mu_{\bar{s}}$, therefore x is also measurable. Moreover

$$\lim_{i \to \infty} \bar{U}(\bar{a}_i) = \int_S x(s) d\mu_{\bar{s}}(s) = \lim_{i \to \infty} \bar{U}(\underline{a}_i).$$

Therefore by bounded convergence theorem of Lebesgue, function $U(x) = \int_S x(s) d\mu_{\bar{s}}(s)$ represents $\succeq_{\bar{s},M}$.

□

Proposition 1 establishes the subjective expected utility representation for the induced preference relation over acts. Notice, that the representation holds only for the acts that are Borel measurable and since I do not restrict A in any way, not all of the acts are. However, I am only interested in the representation for $X \subset A$ and for technical purposes, for simple acts and proposition 1 guarantees that both of these are measurable.

I now turn my attention to the representation of the induced preference relation over menus $\succeq_{\bar{s},a}$. The necessary structure is provided by axioms 11–15.

Axiom 11. *Let $m = \{x\}$, $m' = \{y\}$ and $k \in K$. Then $m \succeq_{K,k} m'$ if and only if $x \succeq_{K,M} y$.*

Axiom 12. *Let $m, m' \in M$ satisfy $m' \subset m$. Then $m \succeq_{K,x} m'$.*

Axiom 13. *Let $m \in M$ and $k \in K$. There exists $m' \subset m$ such that*

$$\forall_{m'' \subset m}(m \succ_{K,k} m'') \iff (m'' \cap m' = \emptyset).$$

Axiom 14. *Let $[\bar{s^*}_K | K \cup \{x\}] = \{\bar{s}_1, ..., \bar{s}_n\}$ and assume that $m \sim_{\bar{s}_i,x} \{x_i\}$. Then $m \sim_{K,x} \{x_0\}$, where $x_0 \in X$ satisfies $x_0 \sim_{K,M} a$ for $a \in A$ defined as*

$$a(s) = \begin{cases} x_1(s), & s \in [\bar{s}_1] \\ \vdots \\ x_n(s), & s \in [\bar{s}_n]. \end{cases}$$

Axioms 11–14 are adapted versions of Cooke (2017) axioms. Axiom 11 states that singleton menus are compared using the induced preference over acts. Axiom 12 is a standard preference for flexibility axiom of Kreps (1979). It states that the larger menu is weakly preferred to the smaller one. Axiom 13 demands that the consumer is evaluating the menu by its best elements. Finally, axiom 14 is a standard rational expectations axiom.

Proposition 2. *Relation $\succeq_K$ satisfy axioms 4 and 11–14 if and only if there exists a probabilistic measure μ_K on a sigma field of Borel subsets of $[\bar{s^*}_K]$ and a continuous, strictly increasing utility function $u : [0,1] \to \mathbb{R}_+$ such that*

$$m \succeq_{K,x} m' \iff E_K \left[\max_{z \in m} \int_{[\bar{s^*}_K]} u(z(s)) d\mu_{K \cup \{x\}}(s) \right] \geq E_K \left[\max_{z \in m'} \int_{[\bar{s^*}_K]} u(z(s)) d\mu_{K \cup \{x\}}(s) \right].$$

Proof. Fix some $x \in X$, $m, m' \in M$ and let $[\bar{s^*}_K | K \cup \{x\}] = \{\bar{s}_1, \ldots, \bar{s}_n\}$. For any $\bar{s}_i$ axiom 13 ensures there exists some x_i, x_i' such that $m \sim_{\bar{s}_i,x} x_i$ and $m' \sim_{\bar{s}_i,x} x_i'$. By axiom 11 and proposition 1 there exists $\mu_{\bar{s}_i}$ such that

$$\{y\} \succeq_{\bar{s}_i,x} \{z\} \iff \int_{[\bar{s^*}_K]} u(y(s))d\mu_{\bar{s}_i}(s) \geq \int_{[\bar{s^*}_K]} u(z(s))d\mu_{\bar{s}_i}(s).$$

Denote $\bar{U}_i(z) = \int_{[\bar{s^*}_K]} u(z(s))d\mu_{\bar{s}_i}(s)$. I now show that x_i, x_i' are maximal elements with respect to $\bar{U}_i$ in m, m' respectively.

Assume that $b \in \operatorname{argmax}_{z \in m} U_i(z)$. By axiom 12 clearly $m \succeq_{\bar{s}_i,x} \{b\}$. Assume that $m \succ_{\bar{s}_i,x} \{b\}$. Then by axiom 13 there exists $b' \in m$ such that $b' \succ_{\bar{s}_i,x} b$ which contradicts definition of b. Therefore $b \sim_{\bar{s}_i,x} m$ and finally $x_i \in \operatorname{argmax}_{z \in m} \bar{U}_i(z)$ and similarly $x_i' \in \operatorname{argmax}_{z \in m'} \bar{U}_i(z)$.

Finally, by axiom 14

$$m \succeq_{K,x} m' \iff E_K\left[\max_{z \in m} \int_{[\bar{s^*}_K]} u(z(s))d\mu_{K \cup \{x\}}(s)\right] \geq E_K\left[\max_{z \in m'} \int_{[\bar{s^*}_K]} u(z(s))d\mu_{K \cup \{x\}}(s)\right].$$

□

The representation of the induced preference over menus is standard. It is the subjective expected value of the best item in the menu. However, before the learning from the first period consumption is realized, it is unknown which item in the menu would be the best, therefore the subjective expected utility from the consumption from the menu is a random variable itself, and the expected value of this variable is calculated. Notice, that the representation for the induced preference over menus is only provided for $\succeq_{K,x}$, meaning that neither hypothetical acts, nor hypothetical knowledge is allowed. It is so, because learning is only defined for the alternatives in X and the realised knowledge of the consumer. I could be more general, but it would unnecessarily complicate notation. Hypothetical alternatives and knowledge are only necessary for the technical purposes, and for the induced preference over acts.

The final assumptions necessary to obtain a joint representation for the conditional preferences, are provided by axioms 15 and 16 below. The first of those axioms is a well known hexagon condition of Debreu (1959) and ensures that the utility of the consumer is separable between the act and the menu, whereas the second one is Karni (2004) uniform utility differences axiom.

Axiom 15. *For $a_1, a_2, a_3 \in A$ and $m_1, m_2, m_3 \in M$ such that $|m_i| = 1$ let $(a_1, m_1) \succeq_{\bar{s}} (a_2, m_2)$ and $(a_2, m_3) \succeq_{\bar{s}} (a_3, m_1)$. Then $(a_1, m_3) \succeq_{\bar{s}} (a_3, m_2)$.*

Axiom 16. *Let $a, a' \in A$ and $y, y' \in X$ satisfy $(a, \{z\}) \succ_K (a', \{z\})$ for all $z \in X$, and $(b, \{y\}) \succ_K (b, \{y'\})$ for all $b \in A$. Then for all $x, x' \in X$, $(a, \{x'\}) \sim_K (a', \{x\})$ if and only if $(a, \{x'_C y\}) \sim_K (a', \{x_C y'\})$ for all $C \subset S$ such that $p_C q \sim_{K,M} q_C p$ for all $p, q \in [0, 1]$.*

Now I am able to state the representation theorem for $\succeq_K$

Theorem 1. *Relation $\succeq_K$ satisfies axioms 4-15 if and only if there exists a scalar $\delta > 0$, together with a probability measure μ_K defined on the sigma field of of Borel subsets of S and a continuous, strictly increasing utility function $u : [0, 1] \to \mathbb{R}_+$ such that function*

$$U(x, m) = \int_{[\bar{s^*}_K]} u(x(s)) d\mu_K(s) + \delta E_K \left[\max_{z \in m} \int_{[\bar{s^*}_K]} u(z(s)) d\mu_{K \cup \{x\}}(s) \right]$$

represents $\succeq_K$, meaning that

$$(x, m) \succeq_K (y, m') \iff U(x, m) \geq U(y, n).$$

Proof. By theorem of Debreu (1959) axiom 15 implies that $\succeq_K$ can be represented by an additive function

$$U(x, m) = u_1(x) + u_2(m),$$

where u_1 represents $\succeq_{K,M}$ and u_2 represents $\succeq_{K,x}$. Therefore

$$U(x, m) = v_1 \left(\int_{[\bar{s^*}_K]} u(x(s)) d\mu_K(s) \right) + v_2 \left(E_K \left[\max_{z \in m} \int_{[\bar{s^*}_K]} u(z(s)) d\mu_{K \cup \{x\}}(s) \right] \right),$$

for some strictly monotone transformations v_1, v_2. Karni (2004) has shown that uniform utility differences as stated in axiom 16 imply that both v_1 and v_2 are affine. Therefore

$$U(x, m) = \int_{[\bar{s^*}_K]} u(x(s)) d\mu_K(s) + \delta E_K \left[\max_{z \in m} \int_{[\bar{s^*}_K]} u(z(s)) d\mu_{K \cup \{x\}}(s) \right],$$

where by the uniqueness part of Debreu (1959) theorem δ is unique.

□

Theorem 1 gives an additively separable representation of the conditional preferences, meaning that the utility from pair (x, m) is the sum of the subjective expected utilities that represent induced preference relations $\succeq_{K,M}$ and $\succeq_{K,x}$. The utility from the consumption from the follow up menu is discounted by scalar δ. Note, that δ does not have to be between 0 and 1. Higher values of δ are allowed, and can be interpreted as the consumer expecting to use the knowledge from the consumption of x more than once.

4 Beliefs and learning

By theorem 1, conditional preferences of the consumer can be represented by a utility function u and a subjective probability measure μ_K. From now on, I always assume that u is an identity function.[2] The main object of study from now on is the subjective probability measure, which I call the beliefs of the consumer.

Representation in theorem 1 is static, meaning that it does not specify how the conditional preferences are updated after the consumption. Definition 1 provides a natural answer to this question, by stating the connection between μ_K and $\mu_{K\cup\{x\}}$.

Definition 1. *Let for all $\bar{s}$ relation $\succeq_{\bar{s}}$ be given. The consumer is bayesian if for all $\bar{s}, \bar{s}'$ such that $\bar{s}' \in [\bar{s}]$*

$$\mu_{\bar{s}'}(C) = \mu_{\bar{s}}(C|\bar{s}') = \frac{\mu_{\bar{s}}(C \cap [\bar{s}'])}{\mu_{\bar{s}}([\bar{s}'])}.$$

Definition 1 is a standard definition of conditional probability, that is the consumer is bayesian if after obtaining new information they update their beliefs in accordance with the definition of a conditional probability. Note, that this definition implies that the consumer correctly anticipates how their beliefs would respond to any new information. Proposition 3 states, that for a bayesian consumer there exists a single probability measure μ defined over a Borel sigma field of S such that each μ_K is obtained from μ by conditioning on $\bar{s^*}_K$.

Proposition 3. *Let $\mu_{\bar{s}}$ for all incomplete preference relations $\bar{s}$ be given. There exists a unique probability measure μ defined over a Borel sigma field of S such that $\mu_{\bar{s}}(C) = \mu(C|\bar{s})$ for all $\bar{s}$ and $C \subset S$ if and only if the consumer is bayesian.*

Proof. Follows from theorem 1 of Kapera (2022) and construction of $\mu_{\bar{s}}$ in proposition 1.

□

Obviously, $\mu = \mu_\emptyset = \mu_{\{x\}}$ for any $x \in X$. Proposition 4 states some basic properties of the beliefs of the consumer.

Proposition 4. *Let $\succeq_{\bar{s}}$ satisfy axioms 4–15. The subjective probability measure $\mu_{\bar{s}}$ which represents $\succeq_{\bar{s}}$ satisfies the following properties.*

[2] Note, that $u \circ \lambda$ is also a Borel measure on X. Therefore I can always demand u to be an identity function (perhaps for a modified measure on X).

1. *(Continuity) Let $x, y \in X$. Then for all $\epsilon > 0$ exists $\delta > 0$ such that $\forall_{z \neq x,y}\ d(x, y) < \delta \implies |\mu_{\bar{s}}([x \succ z]) - \mu_{\bar{s}}([y \succ z])| < \epsilon$.*

2. *(Non-degeneracy) Let $C \subset [\bar{s}]$ be open. Then $\mu_{\bar{s}}(C) > 0$.*

3. *(Restristed indifference) For all $x, y \in X \setminus K$, $\mu_{\bar{s}}([x \sim y]) = 0$*

Proof. All three points of the proposition are obvious. Continuity follows from the continuity of $\succeq_{\bar{s}}$, non-degeneracy from axiom 10 and restricted indifference from axiom 7.

□

I now turn my attention to experimentation and learning behaviour of the consumer. For this part, I need another assumption regarding the beliefs of the consumer.

Definition 2. *Let $x, y \in X$ and denote $C = [x \succ y]$. Probability measure μ_K satisfies learning monotonicity if for all $z \in X \setminus K$*

$$\mu_K([x \succ z]|C) > \mu_K([y \succ z]), \quad \mu_K([x \succ z]|C) > \mu_K([x \succ z]).$$

Learning monotonicity is a very natural property. It states, that learning that $x \succ y$ implies that the consumer updates their beliefs in a way that x is believed to be uniformly better than both x and y were before the update. Assuming this property, I am able to identify the experimental behaviour of the consumer.

Theorem 2. *Let K be given and assume that μ_K satisfies learning monotonicity. Then*

1. *Let $K \neq$. There exists $k \in K$ such that $k \succeq_{K,M} x$ for all $x \in X$.*

2. *For all $x \in X \setminus K$*

$$(x, X) \succeq_K (k, X) \iff \frac{\int_X \mu_K([k \succ z \succ x]) d\lambda(z)}{\int_X \mu_K([x \succ z \succ k]) d\lambda(z)} \leq 1 + \delta.$$

Proof. Let $\bar{U}_K$, U_K represent respectively $\succeq_{K,M}$ and $\succeq_K$. I prove the first point by induction on $|K|$. Let $K = \{k\}$. Since there is no learning from a single alternative, for any $x, y \in X$ consumer beliefs after the consumption of x, y are equal, meaning that $\mu_{\{x\}} = \mu_{\{x\}}$. As such, $\text{argmax}_{x \in X} U(x) = \text{argmax}_{x \in X} \bar{U}(x)$ and $k \in K$ satisfies the first point of the theorem. Now assume, that for an arbitrary K there exists $k \in K$ such that $k \succeq_{K,M} x$ for all $x \in X$ and let $K' = K \cup \{k'\}$. Assume that $U_{K'}(k') > U_{K'}(k)$. Then, by learning monotonicity, for all $x \in X$ $\mu_{K'}([k' \succ x]) > \mu_K([k \succ x])$ and therefore $k' \succ_{K',M} x$. Similarly for $U_{K'}(k) > U_{K'}(k')$, and the proof of the first point of the theorem is finished.

Now fix some knowledge set K, alternative $x \in X \setminus K$ and denote $k = \text{argmax}_{k' \in K} \bar{u}(k')$ where $\bar{u}$ represents $\succeq_{K,M}$. Let $C_1 = [k \succeq x]$, $C_2 = [x \succ k]$, $p_1 = \mu_K(C_1)$, $p_2 = \mu_K(C_2)$ denote

$$x_* = \int_{C_1} x(s) d\mu_K(s), \quad x^* = \int_{C_2} x(s) d\mu_K(s),$$

$$k^* = \int_{C_1} k(s) d\mu_K(s), \quad \int_{C_2} k(s) d\mu_K(s),$$

and define two simple acts a_x, a_k as

$$a_x(s) = \begin{cases} x_*, & s \in C_1 \\ x^*, & s \in C_2 \end{cases}, \quad a_k = \begin{cases} k^*, & s \in C_1 \\ k_*, & s \in C_2 \end{cases}.$$

From representation theorem 1 $k \sim_{K,M} a_k$, $x \sim_{K,M} a_x$ and by learning monotonicity the second period choice from X is equivalent to $k^*_{C_1} x^*$. Therefore

$$(x, X) \succeq_K (k, X) \iff p_1 x_* + p_2 x^* + \delta p_1 k^* + \delta p_2 x^* \geq p_1 k^* + p_2 k_* + \delta p_1 k^* + \delta p_2 k_*.$$

Transforming the inequality above, I obtain

$$(x, X) \succeq_K (k, X) \iff \frac{p_1(k^* - x_*)}{p_2(x^* - k_*)} \leq 1 + \delta.$$

Notice, that by continuity property of proposition 4, both $\mu_K([x \succ z \succ k])$ and $\mu_K([k \succ z \succ x])$ are continuous as functions of z and therefore measurable with respect to λ. Therefore by Fubini–Tonelli theorem

$$p_1(k^* - x_*) = \int_{C_1} k(s) - x(s) d\mu_K(s) = \int_{C_1} \lambda(\{z \in X : k \succ_s z \succ_s x\}) d\mu_K =$$

$$= \int_{C_1} \int_X \mathbb{1}_{\{(z,s) \in X \times C_1 : k \succ_s z \succ_s x\}} d\lambda d\mu_K = \int_X \int_{C_1} \mathbb{1}_{\{(z,s) \in X \times C_1 : k \succ_s z \succ_s x\}} d\mu_K d\lambda =$$

$$= \int_X \mu_K([k \succ z \succ x] \cap C_1) d\lambda(z) = \int_X \mu_K([k \succ z \succ x]) d\lambda(z),$$

and similarly I obtain that $p_2(x^* - k_*) = \int_X \mu_K([x \succ z \succ k]) d\lambda(z)$. Therefore

$$(x, X) \succeq_K (k, X) \iff \frac{\int_X \mu_K([k \succ z \succ x]) d\lambda(z)}{\int_X \mu_K([x \succ z \succ k]) d\lambda(z)} \leq 1 + \delta.$$

□

Theorem 2 states, that the maximal element with respect to the induced preference over acts is always an element of K. Therefore, experimentation is only possible if the discounted expected benefit in the second period consumption from learning some new information is higher than the decrease in the expected utility from first period consumption. Note that the representation of the induced preference over menu in proposition 2 together with learning monotonicity imply that the expected utility from second period consumption always increases after learning.

This result also provides an equivalent condition for the benefit from learning to outweight the decrease in expected utility from first period consumption. This condition states that an $x \in X \setminus K$ is preferred to all the alternatives in K if and only if the average probabilities that $s^* \in [k \succ z \succ x]$ divided by the average probability that $s^* \in [x \succ z \succ k]$ is less than $1+\delta$. The probabilities of $[x \succ z \succ k]$ and $[k \succ z \succ x]$ measure how correlated the beliefs that respectively $x \succ k$, $k \succ x$ are with the beliefs with respect to z, therefore this condition is a restriction on the average correlations of the beliefs, conditionally on the revealed preference between x and k.

From now on, assume that the consumer is bayesian and that subjective probability measure μ is given and constant. I now consider the main question in this article, that is whether preference discovery is possible under market conditions. To answer this question, instead of fixing some knowledge set K I consider a sequence of knowledge sets $(K_i)_{i=0}^{\infty}$ defined as $K_0 = \emptyset$ and $K_{i+1} = K_i \cup \{k_i\}$ where $k_i \in \text{argmax}_{z \in X} U_i(z)$ for U_i being a subjective expected utility representation of $\succeq_{K_i}$. Note, that I assume that $k_i \in X$ without any menu restriction, meaning that the all of the alternatives in X are available at each step. Correspondingly, I always assume that the follow up menu m is equal to X and as such I drop the dependence on menu in the notation, meaning that I write $x \succ_{K_i} z$ instead of $(x, X) \succ_{K_i} (z, X)$.

Definition 3. *Let $(K_i)_{i=0}^{\infty}$ be given. I say that the corresponding sequence of incomplete preferences $(\bar{s^*}_{K_i})_{i=0}^{\infty}$ converges if and only if $\bigcap_{i=0}^{\infty}[\bar{s^*}_{K_i}] = \{s^*\}$.*

Definition 3 states what I consider as the convergence of preferences. This definition formalizes what I understand by preference discovery, meaning that preference discovery takes place if and only if (in the limit) the only preference relation that extends what the consumer has learned is the real preference relation of the consumer. Proposition 5 gives an obvious condition for preference convergence.

Proposition 5. *Sequence of incomplete preferences* $(\bar{s^*}_K)_{i=0}^{\infty}$ *converges if and only if* $K = \bigcup_{i=0}^{\infty} K_i$ *is dense in* X.

Proof. By axiom 2 for any dense subset $K \subset X$ and relation $\bar{s}_K \subset K \times K$ there exists a unique $s \in S$ such that $\bar{s}_K$ is the restriction of s to K. Therefore the condition that K is dense is sufficient. It is also necessary by the non-degeneracy property shown in proposition 4.

□

Obviously, proposition 5 does not answer the question whether it is possible for $K = \bigcup_{i=0}^{\infty} K_i$ to be dense in X. In general, it is not an easy question to answer. However, when μ satisfy learning monotonicity, I am able to answer this question in theorem 3.

Theorem 3. *Denote* $K = \bigcup_{i=0}^{\infty} K_i$ *and let* $\bar{U}$ *be a utility representation of* $\succeq_{K,M}$ *and assume that* μ *satisfies learning monotonicity. There exists open set* $Y \subset X$ *such that* $Y \cap K = \emptyset$.

Proof. Denote $k = \text{argmax}_{x \in X} \bar{U}(x)$ and $k' = \text{argmin}_{x \in X} \bar{U}(x)$. Let $(k_i^*)_{i=1}^{\infty}$ be the sequence of best known alternative, meaning that $k_i^* = \text{argmax}_{k' \in K_i} \bar{U}_i(k')$ where $\bar{U}_i$ represents $\succeq_{K_i,M}$. Obviously if $k_j \in K_j$ for any j then $k_{j'} = k_j$ for $j' > j$ so K is finite and the statement of the theorem holds. Therefore, assume that $k_i \notin K_i$ for all i. Since μ satisfies learning monotonicity, by theorem 2 it implies that

$$\frac{\int_X \mu_{K_i}([k_i^* \succ z \succ k_i])d\lambda(z)}{\int_X \mu_{K_i}([k_i \succ z \succ k_i^*])d\lambda(z)} \leq 1 + \delta.$$

Assume towards the contradiction that K is dense in X. Since X is compact, there exists a convergent subsequence $(k_i')_{i=0}^{\infty}$ of $(k_i)_{i=0}^{\infty}$ such that $k_i' \rightarrow_{i \rightarrow \infty} k'$. Since all elements of $(k_i')_{i=0}^{\infty}$ are different,

$$\forall_i : \quad \frac{\int_X \mu_{K_i}([k_i^* \succ z \succ k_i'])d\lambda(z)}{\int_X \mu_{K_i}([k_i' \succ z \succ k_i^*])d\lambda(z)} \leq 1 + \delta.$$

However

$$\frac{\int_X \mu_{K_i}([k_i^* \succ z \succ k_i'])d\lambda(z)}{\int_X \mu_{K_i}([k_i' \succ z \succ k_i^*])d\lambda(z)} \geq \frac{\mu_{K_i}([k_i^* \succ k_i'])}{\mu_{K_i}([k_i' \succ k_i^*])} \min_{z \in X} \mu_{K_i}([k_i^* \succ z \succ k_i']).$$

By the assumption that K is dense in X and the definition of k'

$$\mu_{K_i}([k_i^* \succ k_i']) \to_{i\to\infty} 1, \quad \mu_{K_i}([k_i' \succ k_i^*]) \to_{i\to\infty} 0,$$

which implies that

$$\frac{\mu_{K_i}([k_i^* \succ k_i'])}{\mu_{K_i}([k_i' \succ k_i^*])} \to_{i\to\infty} \infty,$$

and again by the assumption that K is dense in X

$$\min_{z\in X} \mu_{K_i}([k_i^* \succ z \succ k_i']) \to_{i\to\infty} \min_{z\in X} \mu_K([k \succ z \succ k']) = 1.$$

Therefore

$$\frac{\mu_{K_i}([k_i^* \succ k_i'])}{\mu_{K_i}([k_i' \succ k_i^*])} \min_{z\in X} \mu_{K_i}([k_i^* \succ z \succ k_i']) \to_{i\to\infty} \infty,$$

Which is a contradiction.

□

The obvious conclusion from theorem 3 together with proposition 5 is, that convergence of preferences is impossible. This conclusion is stated by corollary 1.

Corollary 1. *Let μ satisfy learning monotonicity. Then* $\bigcup_{i=0}^{\infty}[\bar{s^*}_{K_i}] \neq \{s^*\}$.

Proof. Follows trivially from theorem 3 and proposition 5.

□

By corollary 1, total preference discovery is not possible without external incentive to experiment. This result does not necessarily mean that the consumer ever ceases to experiment altogether. This is merely a reflection that for sufficiently large K_i the consumer only experiments in some neighbourhood of the best known alternative, and never explores the whole range of the alternatives.

Learning monotonicity is a sufficient condition for both theorem 3 and corollary 1 to hold, but it is not a necessary one. However, without this assumption, the correlations of the consumer beliefs can be arbitrary, and it is not possible to say much about the consumer behaviour. In any case, it is hard to find any reasonable μ for which the preferences converge. Finally, corollary 1 excludes preference convergence on the whole X, when the whole X is available at each stage of the consumption. It does not exclude the possibility, that under some assumptions regarding the sequence of menus, preference convergence is possible. Similarly, it does not exclude the possibility that the preferences converge over some finite subset of X.

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