Existence of insurance with advance information and repeated interactions*

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Abstract

We study the existence of insurance markets with adverse selection problems rooting in individuals' advance information of future idiosyncratic endowment realizations. As the novel feature, we allow for repeated interactions between insurers and insurees. We show that repeated interactions allow the insurer to offer insurance against bad news and to threaten insurees with exclusion from future insurance benefits, which together ease the necessary and sufficient condition for market existence. We find that private unemployment insurance in the US can be profitable for a relatively short exclusion length of one year.

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1 Introduction

Unemployment is one of the most severe risks that many people face. Even with public insurance, it is well documented that becoming unemployed forces people to substantial and undesirable cuts in their consumption expenditures (e.g., Gruber, 1997; Hendren, 2017). In the light of this, why isn't there a thriving private unemployment insurance market in the US? A possible explanation is private advance information. Employing a model originally proposed by Rothschild and Stiglitz (1976), Hendren (2017) argues that individual's foreknowledge about future job losses renders unemployment insurance policies too adversely selected to be profitable.

In the Rothschild and Stiglitz (1976) model, risk-averse agents have private advance information on their idiosyncratic endowment risk. Conditional on this information but before the endowment shock realizes, they decide whether to accept an insurance contract offered by a risk-neutral insurer or to merely consume their endowments in autarky. This decision occurs at a single instance, which is why we refer to it as a single interaction between an insurer and a potential insuree. We develop a dynamic version of this model that allows us to consider repeated interactions between contracting parties. Each period, agents receive a private signal that is informative about their subsequent endowment shock realization and then decide whether to sustain the insurance contract or not. As our main result, we show that repeated interactions relax the necessary and sufficient condition for market existence relative to the single-interaction case.

With repeated interactions, the insurer has the possibility to exclude agents from future insurance in case they decide to default to the no-trade allocation. Exclusion alters agents' incentives to participate in the insurance market. Not only the current but also the future benefits of insurance matter when agents decide about an insurance contract offered to them, which increases their willingness to pay for insurance. The key insight is that the future benefits of insurance are larger than the current benefits considered in a single-interaction model because they include additional insurance possibilities. In particular, the future benefits include insurance against *bad news* received in the future. Therefore, agents can get insurance not only against the fundamental endowment shocks but also against the private news about future shocks. Insurance against bad news cannot be provided in the single-interaction environment because today's news have already been realized.

Intuitively, the longer agents are excluded from future insurance, the higher their willingness to pay for insurance today is, and the more profitable the provision of insurance is to the insurer. Conversely, when insurers can not threaten agents with exclusion from future insurance, the incentives of insurees in our model are identical to those in a single-interaction model, and the future value of insurance is irrelevant for market existence.

Our dynamic model applies to all insurance markets with repeated interactions and private advance information on future realizations of idiosyncratic shocks. These assumptions are particularly well suited for unemployment insurance. First, individual careers entail repeated moves in and out of unemployment. This makes unemployment risk different from other risks such as life or disability risk. Second, Stephens (2004) and Hendren (2017) document that individual subjective job-loss expectations carry significant predictive power for subsequent job losses, even when public information available to an econometrician is taken into account. Thus, individuals possess advance information on their future job losses. Through the lens of our model, we revisit the issue of the missing private unemployment insurance (UI) market in the US and ask: are repeated interactions quantitatively important for the existence of unemployment insurance in the presence of advance information on future job losses?

To address this question, we inform our theoretical model with Hendren (2017)'s estimates of individuals' willingness to pay for unemployment insurance and the costs of adverse selection. Through the lens of his single-interaction model – or equivalently, of our model without exclusion from insurance in the future – his estimates yield costs of adverse selection that by far exceed agents' willingness to pay, hindering the existence of a profitable insurance market.

This changes when we allow for exclusion. Already if agents face the threat to be excluded for one year after defaulting, unemployment insurance can be provided at a profit. Thus, a relatively short exclusion length suffices for a profitable insurance market because the benefits of future insurance are quantitatively important. Thereby, the future benefits of insurance stem predominantly from insurance against bad news that were not accounted for in the previous literature. Correspondingly, our quantitative results imply that knowledge of future job loss alone is unlikely to be the cause of the missing unemployment insurance market and the absence of this market remains a puzzle. Allowing for the possibility to exclude agents from future insurance therefore reopens the debate on the causes of the missing market.

Related literature Hendren (2013, 2017) are the first papers to investigate the importance of private information not only for the functioning but also for the existence of private insurance markets; Hendren (2013) studies the role of private advance information for the existence of insurance markets for long-term care, disability, and life insurance, while Hendren (2017) considers unemployment insurance. We generalize the theoretical results in Hendren (2013, 2017) to repeated interactions, which allows us to highlight the role of exclusion from insurance benefits and insurance against bad news for the existence of profitable insurance markets.

We are not the first to explore the role of market exclusion after a default for consumption insurance in dynamic economies. Bond and Krishnamurthy (2004) point out the pivotal role of exclusion from financial markets for the functioning of unsecured credit markets. Braxton, Herkenhoff, and Phillips (2020) consider a dynamic economy with temporary exclusion from the credit market of defaulting agents. They find public unemployment insurance in the US to be too generous compared to the utilitarian benchmark. While we emphasize the importance of exclusion from future insurance, our focus is on the existence of a profitable UI market with private advance information on future job losses.

Our theoretical approach shares similarities with the literature on the welfare effects of advance information in efficient risk sharing. Hirshleifer (1971) and Schlee (2001) show that advance information can make risk-averse agents ex-ante worse off if such information leads to an evaporation of risks that otherwise could have been shared in a competitive equilibrium with full insurance. Allowing also for the insurance of news as well as fundamental risk, Denderski and Stoltenberg (2020) analyze the social value of better public information when agents have also private advance information about future income shocks. None of these papers study the role of advance information and exclusion for the existence of insurance markets.

In Section 2, we present the model. In Section 3, we provide and discuss our theoretical results on existence of insurance. Section 4 contains a quantitative application of theory to unemployment insurance and the last section concludes.

2 Environment

Time is infinite and there is a unit mass of agents endowed with income y.[1] Each period, agents receive a private signal n about their probability to suffer an income loss l, $0 < l < y$. The shock to their endowment realizes before agents consume. The signal is i.i.d across agents and over time with two realizations, *good* or *bad news*, $n \in \{g, b\}$. When $n = b$, agents incur the loss with probability one.[2] For $n = g$, the probability of a loss is $0 < p < 1$. The probability to receive bad news is $0 < \mu < 1$.

To facilitate comparison with earlier work, we consider contracts that prescribe consumption which depends solely on the current realizations of the signal and the endowment shock. A consumption allocation prescribed by such a contract is denoted by $c = \{c_{gy}, c_{gl}, c_{bl}\}$, with c_{gy} as consumption in case of no loss and $c_{gl}(c_{bl})$ as consumption in the event of a loss after receiving a good (bad) private signal. Thus, the insurance contracts and the corresponding consumption allocations are memoryless.[3]

Agents discount future utilities with $0 < \beta < 1$ and have identical expected-utility preferences over consumption streams. The instantaneous utility function $u : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is strictly increasing, strictly concave and satisfies the Inada conditions. In particular, we define $\tilde{w}(c)$ to be the lifetime expected utility implied by a particular allocation before any risk has been resolved:

$$\tilde{w}(c) \equiv (1-\beta)\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t) = \mu u(c_{bl}) + (1-\mu)\left[pu(c_{gl}) + (1-p)u(c_{gy})\right], \tag{1}$$

Each period after the realization of the private signal, but before the income loss occurs, agents have the option to default to autarky. In case of default, agents consume their endowments and – which is the new element here – are excluded from insurance for $N \geq 0$ periods when they default. The individual rationality constraint of good-signal agents can be compactly written as:

$$\begin{aligned}&(1-\beta)\left[pu(c_{gl}) + (1-p)u(c_{gy})\right] + \beta\tilde{w}(c) \geq \\ &(1-\beta)\left[pu(y-l) + (1-p)u(y)\right] + \beta(1+\beta+...\beta^{N-1})U^{Aut} + \beta^{N+1}(1+\beta+...)\tilde{w}(c),\end{aligned}$$

[1] Opting for an infinite horizon allows us to analytically characterize the necessary and sufficient condition for the absence of trade. In Section 4 we discuss a life-cycle economy, which we consider in detail in the online appendix.

[2] This assumption results in the highest cost of adverse selection.

[3] Considering alternatively insurance contracts that are contingent on the history of endowment shocks and private signals further tightens the no-trade condition, and the non-existence of insurance becomes even less likely than with memoryless contracts. The numerical results for history-dependent contracts are available on request.

or

$$(1-\beta)\left[pu(c_{gl})+(1-p)u(c_{gy})\right]+\beta(1-\beta^N)\tilde{w}(c) \geq$$
$$(1-\beta)\left[pu(y-l)+(1-p)u(y)\right]+\beta(1-\beta^N)U^{Aut},$$

with

$$U^{Aut}=\mu u(y-l)+(1-\mu)\left[pu(y-l)+(1-p)u(y)\right]. \tag{2}$$

With memoryless contracts and i.i.d. private signals, the continuation value $\tilde{w}(c)$ depends only on the future, but not on the current realization of signal and endowment.

For the following, it is convenient to express allocations in terms of utility. Let $C: R \to R^+$ be the inverse of the utility function u. With u strictly increasing and strictly concave, C is strictly convex and strictly increasing. A memoryless utility allocation is then denoted by $h = \{u(c_{gy}), u(c_{gl}), u(c_{bl})\} = \{h_{gy}, h_{gl}, h_{bl}\}$ with the corresponding memoryless consumption allocation as $c = \{C(h_{gy}), C(h_{gl}), C(h_{bl})\}$. Equivalently, we will write $\tilde{w}(h)$ instead of $\tilde{w}(c)$. The set of implementable (or constrained feasible) allocations is defined as follows.

Definition 1 (Implementable allocations) *An allocation* $h = \{h_{gy}, h_{gl}, h_{bl}\}$ *is* implementable *if the following statements hold for all periods* $t \geq 0$.

1. h is resource feasible

$$\mu C(h_{bl})+(1-\mu)[pC(h_{gl})+(1-p)C(h_{gy})] \leq \mu(y-l)+(1-\mu)[p(y-l)+(1-p)y] \tag{3}$$

2. h is incentive compatible

$$(1-\beta)h_{bl}+\beta\tilde{w}(h) \geq (1-\beta)h_{gl}+\beta\tilde{w}(h) \tag{4}$$
$$(1-\beta)\left[ph_{gl}+(1-p)h_{gy}\right]+\beta\tilde{w}(h) \geq (1-\beta)\left[ph_b+(1-p)u(0)\right]+\beta\tilde{w}(h) \tag{5}$$

3. h is individually rational

$$(1-\beta)\left[ph_{gl}+(1-p)h_{gy}\right]+\beta(1-\beta^N)\tilde{w}(h) \geq$$
$$(1-\beta)\left[pu(y-l)+(1-p)u(y)\right]+\beta(1-\beta^N)U^{Aut} \tag{6}$$

$$(1-\beta)h_{bl}+\beta(1-\beta^N)\tilde{w}(h) \geq (1-\beta)u(y-l)+\beta(1-\beta^N)U^{Aut}. \tag{7}$$

Note that the two incentive compatibility constraints simplify to:

$$h_{bl} \geq h_{gl}$$
$$ph_{gl}+(1-p)h_{gy}, \geq ph_{bl}+(1-p)u\left(0\right),$$

which resemble the corresponding conditions in Hendren (2013). The two individual rationality constraints (6)-(7) are different than in his paper when $N > 0$, reflecting repeated interactions in a dynamic economy.

Per construction, the no-trade allocation, $\{u(y), u(y-l), u(y-l)\}$, is implementable. As Hendren (2013), we focus on implementable allocations to study the existence of a profitable insurance market. The question is whether there exists an alternative implementable allocation to autarky that is cost efficient. More formally, such an allocation is defined as follows.

Definition 2 (Cost-efficient allocation) *A cost-efficient allocation h^* is implementable and maximizes the slack on the resource constraint:*

$$h^* = \arg\max_{h_{bl}, h_{gl}, h_{gy}} [(1-\mu)p + \mu](y-l) + (1-\mu)(1-p)y \\ - \mu C(h_{bl}) - (1-\mu)[pC(h_{gl}) + (1-p)C(h_{gy})]. \tag{8}$$

A cost-efficient allocation is the allocation that a profit-maximizing monopolist insurer chooses whose choices are constrained by individual rationality and incentive constraints.[4]

3 Analysis

In this section, we deliver our main theoretical result on the necessary and sufficient no-trade condition with repeated interactions and a fixed length of exclusion.

3.1 Existence of insurance

As an intermediate step before the main theoretical result, we characterize cost-efficient allocations with trade, for which any two elements differ from autarky. We show that if such allocations exist, they feature insurance, which creates slack on the resource constraint.

Lemma 1 (Cost-efficient allocation with trade) *Let $h^* = \left\{h^*_{gl}, h^*_{bl}, h^*_{gy}\right\}$ be a cost-efficient allocation with trade. Then, the following statements hold.*

(i) Incentive constraints of bad-signal agents hold with equality, so that the utility of agents who incur an income loss is equalized across signal realizations, $h^*_{bl} = h^*_{gl} = h^*_l$, and the incentive constraints of good-signal agents are slack.

(ii) Individual rationality constraints of good-signal agents hold with equality, the individual rationality constraints of bad-signal agents are slack.

(iii) h^* is characterized by $u(y-l) < h^*_l \leq h^*_{gy} < u(y)$.

The exact proof follows standard arguments and is provided in the online appendix. The logic of the proof can be summarized as follows. For part (i), agents with good signals have a higher outside option value than the bad-signal agents which is reflected in the cost-efficient allocation due to individual rationality constraints, and so their expected utility exceeds the one of the bad-signal agents. Thus, only agents with a bad signal have an incentive to report a good signal realization but not vice versa. For part (ii), convexity of resource costs implies that

[4] The online appendix to Hendren (2013) offers a formal discussion of the maximization problem. The question of insurance market existence boils down to whether a monopolist insurer can incur profits or not. This test of existence circumvents the issue of potentially non-existent competitive Nash equilibria.

it is optimal to equalize the consumption of all agents who suffer an income loss. Ideally, the cheapest way to satisfy the incentive compatibility constraints is to deliver perfect insurance, setting $h_{gy} = h_l$, but this might be prevented by the individual rationality constraint of good-signal agents. Agents with a bad signal benefit from the insurance contracts directly by receiving transfers from fortunate agents and indirectly because with insurance their continuation value is higher than without trade. These two benefits together render the individual rationality constraints of bad-signal agents slack. Lemma 1 implies that the problem to compute a cost-efficient allocation simplifies to:

$$\max_{h_{gy},h_l} [(1-\mu)p+\mu](y-l)+(1-\mu)(1-p)y \\ -[\mu+(1-\mu)p]\,C(h_l)-(1-\mu)(1-p)C(h_{gy}), \tag{9}$$

subject to individual rationality constraints with that of the good-signal agents met with equality. As autarky is implementable and yields zero profits, insurers have no incentive to choose an allocation that requires more resources, implying that an optimal allocation satisfies resource feasibility, though not necessarily with strict equality. We proceed to our main theoretical result and show under which conditions autarky is the only implementable allocation and therefore also the cost-efficient allocation.

Theorem 1 (No Trade) *The autarky allocation $h = \{u(y), u(y-l), u(y-l)\}$ is the only implementable allocation if, and only if*

$$\frac{u'(y-l)}{u'(y)} \leq \underbrace{T_s(p,\mu)T_r(p,\mu,\beta,N)}_{T_d(p,\mu,\beta,N)} \tag{10}$$

with $T_s(p,\mu)$ as the single-interaction pooled price ratio:

$$T_s(p,\mu) = \frac{\mu+(1-\mu)\,p}{(1-\mu)\,p} = \frac{\mathbb{E}\,[P|P\geq p]}{1-\mathbb{E}\,[P|P\geq p]}\frac{1-p}{p}$$

and with $T_r(p,\mu,\beta,N)$ as diminishing factor resulting from repeated interactions

$$T_r(p,\mu,\beta,N) = \left\{\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^N)\mu}\right\}, \quad 0 < T_r \leq 1$$

and $0 < T_d(p,\mu,\beta,N) \leq T_s(p,\mu)$ the repeated-interactions pooled price ratio.

The proof is provided in Appendix A. This condition generalizes earlier findings by allowing for repeated interactions between insurers and agents. For $N = 0$, the condition reduces to

$$\frac{u'(y-l)}{u'(y)} \leq \frac{\mathbb{E}\,[P|P\geq p]}{1-\mathbb{E}\,[P|P\geq p]}\frac{1-p}{p}, \tag{11}$$

with $u'(y-l)/u'(y)$ as agents' willingness to pay for insurance, resembling the condition provided in Hendren (2013, 2017) for a single interaction. While $N = 0$ is also a possible choice with an infinite horizon, it is not necessarily a natural choice because agents evaluate allocations according to their lifetime utility (1).

The main message from Theorem 1 is that the no-trade condition becomes more restrictive for $N > 0$, that is, it is more likely that autarky is *not* the only implementable allocation. With $N > 0$, opting for autarky becomes costlier because agents sacrifice a part of the future benefits of insurance. This can be seen by re-arranging the no-trade condition as follows:

$$\underbrace{\left[\frac{1}{T_r(p,\mu,\beta,N)}\right]}_{\geq 1} WTP \leq T_s(p,\mu).$$

On the left-hand side, the willingness to pay increases due to repeated interactions because $T_r(p,\mu,\beta,N) \geq 1$ is decreasing in N. The term on the right-hand side, $T_s(p,\mu)$, summarizes the costs of adverse selection.

More specifically, the tightness of the no-trade condition depends on the appeal of future insurance benefits to good-signal agents. Thereby, the increased attractiveness of the insurance contract with repeated interactions does not simply stem from extending the benefits in case of a single interaction to multiple periods but also from additional insurance possibilities. First, with repeated interactions, there can be insurance even in case of $p = 0$, that is, there is insurance against *bad news* in the future. This type of insurance cannot be provided with a single interaction because the signals have already been realized and can therefore no longer be insured, resulting in infinitely large T_s. With repeated interactions, however, the limiting value of T_d for arbitrarily small p is finite and given by

$$\lim_{p\to 0} T_d(p,\mu,\beta,N) = \frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]}{(1-\mu)\beta(1-\beta^N)} > 1.$$

Second, insurance is more relevant for good-signal agents because it becomes more likely they need it. For one period, the probability of a good-signal agent suffering an income loss l is p, which constitutes the risk she likes to insure away. For N future periods, the probability for an agent who always receives a good signal to suffer an income loss at least once is $\text{Prob}(l) = 1-(1-p)^{N+1}$. Thus, in the infinite limit the probability to incur a loss at least once is one, which increases the attractiveness of insurance for good-signal agents.

3.2 No-trade theorem with repeated interactions: discussion

Theorem 1 considers private i.i.d. signals on future income losses for a given length of exclusion from the contract. In the following, we begin with studying how the no-trade condition is affected when signals are persistent and then consider random instead of determinsitic return from autarky as in the literature on sovereign default (e.g. Arellano, 2008) or search for credit (e.g. Braxton et al., 2020). Afterwards, we investigate the case of public instead of private signals to ask whether individual rationality alone can limit insurance possibilities.

Persistent signals In the model, the probability to receive a particular signal n' tomorrow does not depend on the signal realization n today, $\text{Prob}(n'|n) = \text{Prob}(n')$. Lemma 1 implies that the no-trade condition is primarily driven by the individual rationality constraints of good-signal agents. Therefore, depending on which signal becomes persistent, there are opposite effects on the no-trade condition. Consider first persistent good signals, $\text{Prob}(n' = g|n = g) > \text{Prob}(n' = g|n = b) = 1 - \mu$. Then, insuring against bad news becomes less important for good signal agents. Correspondingly, their willingness to pay for insurance decreases, rendering no-trade more likely. Assume alternatively that bad signals become persistent, $\text{Prob}(n' = b|n = b) > \text{Prob}(n' = b|n = g) = \mu$. Now the willingness of good-signal agents to pay for insurance increases because receiving a bad signal once has long-lasting negative consequences, making the insurance of bad news more relevant and the existence of insurance more likely.

Random return after default After a default, agents in our environment are allowed back into insurance for sure after being excluded for N periods. Assume alternatively that returning from autarky is possible every period but random with $0 \leq \theta \leq 1$ as the probability to return, implying that the expected time until re-entry (including the current period) is given by $1/\theta$.[5] We show that the no-trade condition has an equivalent structure to Theorem 1 and is given by

$$\frac{u'(y-l)}{u'(y)} \leq T_s(p,\mu)T_r(p,\mu,\beta,\theta), \tag{12}$$

with the single-interaction pooled price ratio $T_s(p,\mu)$ as in Theorem 1 and

$$T_r(p,\mu,\beta,\theta) = \frac{p\left[1-\beta(1-\theta)\mu\right]}{p\left[1-\beta(1-\theta)\mu\right]+\beta(1-\theta)\mu}.$$

Ceteris paribus, the higher is the probability to return, the more likely it is that autarky is the only implementable allocation. Observe that $T_r(p,\mu,\beta,N=0) = T_r(p,\mu,\beta,\theta=1) = 1$ and $\lim_{N\mapsto\infty} T_r(p,\mu,\beta,N) = T_r(p,\mu,\beta,\theta=0)$. Furthermore, there is an explicit relationship between N and θ that yields an identical no-trade condition:

$$\theta = \frac{\beta^N(1-\beta)}{1-\beta^{N+1}}.$$

Can individual rationality alone limit insurance possibilities? Our main theorem reveals that individual rationality and private information together restrict insurance possibilities. To clarify the importance of both frictions, we show that individual rationality alone does not prevent the existence of insurance in our environment. Suppose alternatively that agents receive *public* signals on their income-loss probability that have the same properties as the private ones. The cost-efficient allocation maximizes the slack on resource constraints subject to implementability given now only by resource feasibility and individual rationality. In the following proposition, we show that there always exists insurance.

Proposition 1 (Public information) *Consider a cost-efficient allocation with public signals. Autarky is not the cost-efficient allocation.*

The proof is provided in the Online Appendix. Any insurance transfers between good-signal agents increases their lifetime utility. The increase in lifetime utility also increases the continuation value of bad-signal agents and thus indirectly their lifetime utility. Thus, individual

[5] For a detailed description of random return, we refer to the Online Appendix.

rationality constraints of bad-signal agents are satisfied even when they receive no transfers from good-signal agents. With public signals, incentive constraints are absent, rendering such a transfer also implementable. With private information, such a transfers scheme would violate the incentive constraints of bad-signal agents. As a consequence, individual rationality alone cannot explain the absence of a profitable private insurance market.

4 Quantitative example: private unemployment insurance

As an application of Theorem 1, we study the quantitative importance of repeated interactions and the length of exclusion for the existence of a private unemployment insurance market in the US. As our main result, we find that repeated interactions matter for the existence of unemployment insurance. In particular, we discover unemployment insurance to be profitable already for a relatively short length of exclusion.

4.1 Quantitative exercise: estimates and calibrated parameters

We begin with reviewing estimates for the willingness to pay for unemployment insurance and the pooled-price ratio in the US. Afterwards, we describe how we calibrate the structural parameters of the model presented in the previous section to be consistent with these estimates.

We target Hendren (2017)'s annual estimates of the willingness-to pay (WTP) from the Panel Study of Income Dynamics (PSID) as well as the pooled price ratio (PPR) and the mean of the subjective job loss probability distribution based on the Health and Retirement Survey (HRS), $\text{Prob}(U)$, which are summarized in Table 1.

The willingness to pay estimates – for relative risk aversion $\sigma = 1, 2, 3,$ – one-for-one match the ratio of marginal utilities in the model. The pooled-price ratio in the model from the previous section, $T_s(p, \mu)$, is parameterized by p and μ. To identify these parameters, we employ the estimate of the pooled-price ratio and the mean of the subjective job loss probability distribution. In the model, there are two subjective unemployment probabilities, $\text{Prob}(U|n)$, conditional on the signal n. Thus, the mean of the job loss probability in the model is

$$\begin{aligned} \text{E}[\text{Prob}(U|n)] &= \text{Prob}(U|n = g)\,\text{Prob}(n = g) + \text{Prob}(U|n = b)\,\text{Prob}(n = b) \\ &= p(1 - \mu) + \mu, \end{aligned} \tag{13}$$

and we choose $\{\mu, p\}$ to solve

$$\text{E}[\text{Prob}(U|n)] \overset{!}{=} \text{Prob}(U) = 0.031 \tag{14}$$

$$T_s(\mu, p) \overset{!}{=} \text{PPR} = 4.36, \tag{15}$$

which results in $p = 0.0073$ and $\mu = 0.0239$. For the discount factor, we choose a standard annual value of $\beta = 0.96$, implying an annual real interest rate of four percent. The estimate of the pooled price ratio by far exceeds the willingness to pay in all three cases, clearly satisfying the no-trade condition for a single interaction (11). Correspondingly, Hendren (2017) concludes that unemployment insurance contracts in the US would be too adversely selected to be profitable.

Table 1: Estimates and calibrated parameters

	Estimate/Parameter	Value
WTP	Willingness to pay	$\{1.29, 1.58, 1.87\}$
PPR	Pooled price ratio, $\inf[T_s]$	4.36
Prob(U)	Mean, subjective job loss probability distribution	0.031
μ	Measure of bad-signal agents	0.0239
p	Unemployment probability good-signals agents	0.0073
β	Annual discount factor	0.96

Notes: WTP for unemployment insurance with relative risk aversion $\sigma = \{1, 2, 3\}$. Pooled price ratio as point estimate for the minimum pooled price ratio (semi-parametric), $\inf[T_s]$, evaluated at the mass point Prob$(U) = 0.031$.

4.2 Exclusion and the existence of unemployment insurance

To assess the importance of repeated interactions of insurer and insuree versus a single interaction, we compute the number of exclusion periods necessary for the existence of unemployment insurance in the US. In the next step, we compare the computed exclusion periods to exclusion periods observed in reality.

Given parameter values for p, μ, β, we compute the lowest number of exclusion periods needed for the existence of unemployment insurance, N_{min}, as follows:

$$N_{min} \equiv N : \frac{u'(c_u)}{u'(c_e)} - T_d(p, \mu, \beta, N) = 0.$$

The resulting values are reported in the first row of Table 2. Depending on risk aversion, the minimum total number of exclusion periods for the existence of unemployment insurance varies between less than one year for $WTP = 1.87$ to approximately two and a half years for $WTP = 1.29$.[6] Why does a relatively short exclusion length suffice to overturn Hendren (2017)'s results? The probability to receive a good signal and become unemployed is relatively small with $(1 - \mu)p = 0.0071$ but the probability to receive bad news and become unemployed, $\mu \times 1 = 0.0239$, is over three times higher. Thus, the benefits from insurance stem predominantly from insurance against bad news, a type of insurance that is absent by construction in Hendren (2017)'s single-interaction model.

In the second row of Table 2, we also display the expected number of total exclusion periods when return is random. Comparing the exclusion periods in the first and second row, we find that the expected length of exclusion is slightly larger than in case of a fixed-length exclusion $N_{\min}$. The reason for this is as follows. When return from autarky is random, there is a non-zero probability to return from autarky earlier than with a deterministic exclusion length, making autarky more attractive for the agents. To compensate for this and to render insurance more attractive than autarky, the expected length of exclusion must be larger than in the deterministic case. These quantitative results imply that for a total (expected) length of exclusion of up to three years, private unemployment insurance could be provided at a profit.

[6] With insurance contracts that are additionally contingent on the employment state and the signal realization in the previous period, an exclusion of a single year also suffices for existence not only in case of $\sigma = 3$ but also when $\sigma = 2$, for deterministic as well as for random exclusion.

Is the required length of the exclusion period long or short? Arguably, there is no thriving private unemployment insurance market in the US, which makes it difficult to say whether the number of exclusion periods in Table 2 are realistic. To put the numbers into perspective, we compare them to two real-world analogues: the loss in financial liberty resulting from *private bankruptcy* and the exclusion time after a *sovereign default*.

In the US, individuals can file for *private bankruptcy* according to Chapter 7 (about 71% of filings) or Chapter 13 (29% of filings). In both cases, the bankruptcy appears on the individual's credit history for a given time period with the consequence that it is either impossible to receive unsecured credit or only possible at a high interest rate premium. The loss of access to financial markets closely resembles the idea of exclusion from insurance. The private bankruptcy entry appears in the individual credit history for ten (Chapter 7) or seven years (Chapter 13). In the light of theses numbers, a necessary exclusion length of up to three years do not appear to be unrealistically large.

Another possibility is to compare the exclusion numbers to the time until countries gain re-access to international financial markets after a *sovereign default*. For example, Schmitt-Grohé and Uribe (2017) in Chapter 13 provide estimates how long a sovereign default lasted on average for a sample of countries in the years 1974–2014. Excluding the period of default, it takes on average about 9 years until countries can borrow again some amount (partial re-access) and about 15 years until they can borrow an amount exceeding 1% of their GDP (full re-access). Thus, both estimates are well above the up to three years of exclusion periods necessary for the existence of profitable unemployment insurance.

To gain more confidence in the computed exclusion lengths, we also discuss the relevance of a finite instead of an infinite planning horizon of the agents for the exclusion length necessary to provide insurance at a profit.

Finite planning horizon With an infinite horizon, an exclusion length of up to three years is found be enough to generate slack on the resource constraint. A natural question is whether this conclusion changes when agents' planning horizon is shorter than three years, for example, because of retirement. This is a relevant question because the employed individuals in Hendren (2017) HRS sample are up to 64 year old. The (full) retirement age in the US is 67, which makes considering a finite planning horizon relevant, at least for some of the individuals in the HRS sample. For this reason, we study a life-cycle economy. We provide the details on this version of the model in the online appendix, focussing on key takeaways here.

As in the infinite horizon economy, we consider deterministic and random exclusion. Therefore, a particular given exclusion length for all age groups can not be longer than the remaining time until retirement. For example, a total exclusion length of two periods is only relevant for all agents that have at least two years until they reach the full retirement age. An important feature of the life-cycle economy is that the insurer can condition insurance premia and benefits on age, allowing for inter-generational transfers. For this economy, we vary the total exclusion length and compute the slack on resource constraint.

We find very similar total exclusion lengths for the finite horizon economy as the ones computed in Table 2. With deterministic exclusion, an exclusion of one year is enough for

Table 2: Existence of insurance: minimum number of exclusion periods

	WTP, $u'(c_u)/u'(c_e)$, for various σ		
	1.29 ($\sigma = 1$)	1.58 ($\sigma = 2$)	1.87 ($\sigma = 3$)
Fixed exclusion, N_{min}	2.6803	1.1774	0.6962
Random exclusion, $\mathbb{E}[N_{min}]$	2.8906	1.2309	0.7207

Notes: Minimum number of future exclusion periods for the existence of unemployment insurance, N_{min}, and expected future exclusion length $\mathbb{E}[N_{min}] = 1/\theta - 1$ as functions of the willingness to pay, WTP.

all willingness to pay estimates to provide insurance at a profit. For the random exclusion specification, the expected exclusion, $\mathbb{E}[N_{min}]$, ranges from less than one year for $\sigma = 3$ to 2.7 years in case of log utility.

There are two opposing effects on the resource constraint in this version of the model. On the one hand, the individual rationality constraints prevent the exclusion of agents that are close to retirement. On the other hand, agents that are far from retirement value insurance highly which allows the insurer to extract relatively more resources from these agents. In our quantitative results, the latter dominates the former effect.

5 Conclusions

The main takeaway from our analysis is that the future benefits of insurance matter for market existence. These benefits naturally emerge when insurer and insuree meet repeatedly as in case of unemployment insurance. We find these benefits to be sizeable such that a relatively small number of exclusion periods suffices to render the UI market profitable. Considering the future benefits, adverse selection alone is unlikely to explain the non-existing unemployment insurance market.

Our analysis implies that a relevant avenue for future research is to carefully measure the future value of insurance that results from repeated interactions between an insurer and an insuree. While exclusion as our focus determines whether the value is reflected in insurance contracts or market existence conditions, the size of the future benefits of insurance depends on several factors. A non-exhaustive list includes how persistent employment, unemployment and news about job losses are. Unlike in the single-interaction environment, the dynamic effects of moral hazard can now matter for the existence of insurance. We leave the quantification of their importance for the missing UI market for future work.

A Proof of Theorem 1

The proof proceeds in two parts. First, we show that the no-trade condition is sufficient. In the second part, we show the no-trade condition is also necessary.

Part I: No-trade condition is sufficient We show that if the no-trade condition holds, autarky is the cost-efficient allocation. From Lemma 1, we consider the problem of choosing $\{h_{gy}, h_l\}$ to maximize (9) subject to the individual rationality constraints of good-signal agents

(6):

$$\left[(1-\beta)p+\beta(1-\beta^N)\left[\mu+(1-\mu)p\right]\right]h_l \\ +\left[(1-\beta)(1-p)+\beta(1-\beta^N)(1-\mu)(1-p)\right]h_{gy}\geq U^{Aut}(g)$$

The first-order conditions for h_{gy}, h_l are

$$(1-\mu)(1-p)C'(h_{gy})=\lambda_g^{IR}\left[(1-\beta)(1-p)+\beta(1-\beta^N)(1-\mu)(1-p)\right]$$
$$\left[\mu+(1-\mu)p\right]C'(h_l)=\lambda_g^{IR}\left[(1-\beta)p+\beta(1-\beta^N)\left[\mu+(1-\mu)p\right]\right],$$

with $\lambda_g^{IR}\geq 0$ as the multiplier on the constraint (6). The two first order conditions can be combined into a single first order condition

$$\frac{\mu+(1-\mu)\,p}{(1-\mu)\,p}\left\{\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^N)\mu}\right\}=\frac{C'(h_{gy})}{C'(h_l)}=\frac{u'\left(c_l\right)}{u'\left(c_{gy}\right)}.$$

Next, if the no-trade condition holds, and using (10), it follows

$$\frac{u'\left(y-l\right)}{u'\left(y\right)}\leq\frac{\mu+(1-\mu)\,p}{(1-\mu)\,p}\left\{\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^N)\mu}\right\}=\frac{u'\left(c_l\right)}{u'\left(c_{gy}\right)}.$$

From Lemma 1, we have $c_l\geq y-l$ and $c_{gy}\leq y$. With decreasing marginal utility, this is only possible if $c_l=y-l$ and $c_{gy}=y$. Hence, when the no-trade condition holds, the cost-efficient allocation is the no-trade allocation.

Part II: No-trade condition is necessary We show that if the no-trade condition is violated, there is an implementable allocation with trade which generates a slack in the resource constraint. Without loss of generality, consider a perturbed allocation $h=\{h_{gy}=u(y-\delta),h_l=u(y-l+\gamma)\}$ with $\delta>0,\gamma>0$ with δ,γ arbitrarily small. By definition, this is an allocation with trade which also satisfies all other results in Lemma 1, but we don't know if this perturbed allocation is resource feasible. Net resources saved by this perturbation are $(1-\mu)\,(1-p)\,\delta-\left[\mu+(1-\mu)p\right]\gamma$. Observe that when this is non-negative, this perturbed allocation is implementable. By Lemma 1, the allocation implies the following pattern of binding individual rationality constraints

$$\tilde{w}(g)^N=U^{Aut}(g)^N, \tag{16}$$
$$\tilde{w}(b)^N>U^{Aut}(b)^N. \tag{17}$$

Before the signal realizes, the relevant expected utilities for N future periods are

$$\tilde{w}^N=(1-\mu)\tilde{w}(g)^N+\mu\tilde{w}(b)^N$$
$$U^{Aut,N}=(1-\mu)U^{Aut}(g)^N+\mu U^{Aut}(b)^N$$

Let $\tilde{w}(g), \tilde{w}(b)$ and $U^{Aut}(g), U^{Aut}(b)$ be the lifetime utilities of good-and bad-signal agents as the corresponding limiting expressions $N \to \infty$ of both sides of the individual rationality constraints (16) and (17). The following relationships between lifetime utilities and the period-N utilities apply

$$\tilde{w}(g)^N = \tilde{w}(g) - \beta^{N+1}\tilde{w} \tag{18}$$
$$\tilde{w}(b)^N = \tilde{w}(b) - \beta^{N+1}\tilde{w} \tag{19}$$
$$U^{Aut}(g)^N = U^{Aut}(g) - \beta^{N+1}U^{Aut} \tag{20}$$
$$U^{Aut}(b)^N = U^{Aut}(b) - \beta^{N+1}U^{Aut}. \tag{21}$$

Let ϵ be the slack on the individual rationality constraint of bad-signal agents:

$$\tilde{w}(b)^N - U^{Aut}(b)^N = \epsilon > 0.$$

Individual rationality constraints of good-signal agents are binding, $\tilde{w}(g)^N - U^{Aut}(g)^N = 0$. Using (19), (21) as well as (18), (20), gives the following restrictions

$$\tilde{w}(b)^N - U^{Aut}(b)^N = \epsilon = \tilde{w}(b) - U^{Aut}(b) + \beta^{N+1}(U^{Aut} - \tilde{w})$$
$$\tilde{w}(g)^N - U^{Aut}(g)^N = 0 = \tilde{w}(g) - U^{Aut}(g) + \beta^{N+1}(U^{Aut} - \tilde{w}),$$

Using the definition of lifetime utilities of good and bad-signal agents in the insurance contract and in autarky, one eventually gets

$$\tilde{w} - U^{Aut} = \frac{\mu\epsilon}{1-\beta^{N+1}}.$$

For the individual rationality constraint of the bad-signal agents (17) combined with (19) and (21), it follows that:

$$\tilde{w}(b)^N - U^{Aut}(b)^N = \epsilon = (1-\beta)u'(y-l)\gamma + \beta(1-\beta^N)\frac{\mu\epsilon}{1-\beta^{N+1}}$$

such that ϵ can be written as

$$\epsilon = \frac{(1-\beta)\left(1-\beta^{N+1}\right)\gamma}{1-\beta^{N+1}-\beta(1-\beta^N)\mu}u'\left(y-l\right). \tag{22}$$

For the binding individual rationality constraint of good-signal agents (16) we have:

$$0 = \tilde{w}\left(g\right)^N - U^{Aut}\left(g\right)^N = (1-\beta)\left[-\left(1-p\right)u'\left(y\right)\delta + pu'\left(y-l\right)\gamma\right] + \beta(1-\beta^N)\frac{\mu\epsilon}{1-\beta^{N+1}}$$

This implies:

$$\frac{\left(1-\mu\right)\left(1-p\right)\delta}{\left[\mu+(1-\mu)p\right]\gamma} = \frac{u'\left(y-l\right)}{u'\left(y\right)}\frac{\left(1-\mu\right)p}{\mu+\left(1-\mu\right)p}\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^N)\mu}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}. \tag{23}$$

Suppose that the no-trade condition is violated:

$$\frac{u'(y-l)}{u'(y)} > \frac{\mu+(1-\mu)\,p}{(1-\mu)\,p}\left\{\frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p+\beta(1-\beta^{N})\mu}\right\}.$$

Using this, (23) boils down to:

$$\frac{(1-\mu)\,(1-p)\,\delta}{[\mu+(1-\mu)p]\,\gamma} > 1 \implies (1-\mu)\,(1-p)\,\delta-[\mu+(1-\mu)p]\,\gamma > 0,$$

thus, net resources are positive, and the no-trade condition is not only sufficient but also necessary.

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B Online Appendix

B.1 Proof of Lemma 1

We prove this lemma in several steps. To begin with, we show all implementable allocations must at least deliver the expected lifetime utility of autarky.

Step 1 *Let h be an implementable allocation. The following statements hold*

(i) Allocation h delivers a lifetime expected utility not worse than that of autarky, $\tilde{w}(h) \geq U^{Aut}$.

(ii) It is $\tilde{w}(h) = U^{Aut}$ if, and only if, h is the no-trade (autarky) allocation.

(iii) Let h be an implementable allocation with trade. Then, $h_{bl} \geq u(y-l)$ and the individual rationality constraint of the bad-signal agents is slack.

Proof. These statements are standard results. For this reason, we just sketch the proofs and omit the details. If (i) was not true, adding the two individual rationality constraints would result in h being not implementable. The if part of statement (ii) is by definition. For the only-if part, setting $\tilde{w}(h) = U^{Aut}$ collapses the individual rationality constraints to current period utilities which $\tilde{w}(h)$ and U^{Aut} are averages of (with identical weights). Thus, both individual rationality constraints can only be satisfied if $h_{gl} = h_{bl} = u(y-l)$ and $h_{gy} = u(y)$ to also meet $\tilde{w}(h) = U^{Aut}$. Hence, with trade it must be that $\tilde{w}(h) > U^{Aut}$. Statement (iii) is proven by contradiction, assuming $h_{bl} < u(y-l)$ together with (4) and (6) implies $h_{gl} < u(y-l)$ and $h_{gy} > u(y)$ which either violates resource feasibility or yields $\tilde{w}(h) < U^{Aut}$. ■

In the following step, we prove Part (i) of Lemma 1.

Step 2 *Let h be a cost-efficient allocation with trade. Incentive constraints of bad-signal agents hold with equality, so that utility for agents who incur an income loss is equalized across signal realizations, $h_{bl} = h_{gl} = h_l$, and the incentive constraints of good-signal agents are slack.*

Proof. Given the incentive compatibility constraint of bad-signal agents, we only need to consider implementable allocations with trade which have either $h_{bl} > h_{gl}$ or $h_{bl} = h_{gl} = h_l$.

For the first case, consider a cost-efficient allocation with trade, $h_0 = \{h_{gy}, h_{gl}, h_{bl}\}$, $h_{bl} > h_{gl}$. Thus, by assumption, h_0 is also implementable (satisfies all constraints). The resulting expected utility is:

$$\tilde{w}(h_0) = \underbrace{(1-\mu)(1-p)h_{gy}}_{\tilde{w}_y(h_0)} + \underbrace{(1-\mu)ph_{gl} + \mu h_{bl}}_{\tilde{w}_l(h_0)}$$

with $\tilde{w}_y(h_0)$ as the utility of agents without and $\tilde{w}_l(h_0)$ as the expected utility of agents with income loss. The resource costs $\tilde{C}(h_0)$ of this allocation are:

$$\tilde{C}(h_0) = \underbrace{(1-\mu)(1-p)C\left(h_{gy}\right)}_{\tilde{C}_y(h_0)} + \underbrace{(1-\mu)pC\left(h_{gl}\right) + \mu C\left(h_{bl}\right)}_{\tilde{C}_l(h_0)}$$

Next, let ε_b^{ir} be the utility slack in the bad-signal agents individual rationality constraint (7) and $\varepsilon_b^{ic} = h_{bl} - h_{gl}$ the utility slack in the incentive constraint of bad-signal agents (4). By assumption, $\varepsilon_b^{ir} > 0$ and $\varepsilon_b^{ic} > 0$. Consider a perturbation $h_1 = \{h_{gy}, h_{gl} + \delta_g, h_{bl} - \delta_b\}$ such that $\delta_g = \frac{\mu\delta_b}{(1-\mu)p}$ and the expected utility of the alternative allocation is given by

$$\tilde{w}(h_1) = \underbrace{(1-\mu)(1-p)h_{gy}}_{\tilde{w}_y(h_1)} + \underbrace{(1-\mu)p\,(h_{gl}+\delta_g) + \mu\,(h_{bl}-\delta_b)}_{\tilde{w}_l(h_1)} = \tilde{w}(h_0).$$

This perturbation keeps the incentive compatibility of good-signal agents satisfied (as it was already met by h_0, implementable by assumption, and now we are decreasing h_{bl} and increasing h_{gl}). It also trivially satisfies the good-signal agents individual rationality constraint (the continuation value $\tilde{w}(h_1) = \tilde{w}(h_0)$ is unchanged and we have increased h_{gl}). We need to ensure that the remaining two constraints are also satisfied. The individual rationality constraint of bad-signal agents is satisfied for $\delta_b \leq \varepsilon_b^{ir}/(1-\beta)$. The incentive compatibility constraint of bad-signal agents requires:

$$h_{bl} - \delta_b \geq h_{gl} + \delta_g \iff \delta_b \leq \varepsilon_l^{ic} \frac{(1-\mu)p}{(1-\mu)p+\mu}$$

Thus, $\delta_b \leq \min\left\{\frac{\varepsilon_l^{ir}}{1-\beta}, \varepsilon_l^{ic}\left[\frac{(1-\mu)p}{(1-\mu)p+\mu}\right]\right\}$ ensures that the perturbed allocation is implementable. The perturbed allocation requires resource costs $\tilde{C}(h_1)$:

$$\tilde{C}(h_1) = \underbrace{(1-\mu)(1-p)C\,(h_{gy})}_{\tilde{C}_y(h_1)} + \underbrace{(1-\mu)pC\,(h_{gl}+\delta_g) + \mu C\,(h_{bl}-\delta_b)}_{\tilde{C}_l(h_1)}$$

The difference in required resources, for an arbitrarily small δ_b is

$$\begin{aligned}\tilde{C}(h_1) - \tilde{C}(h_0) &= (1-\mu)p\,[C\,(h_{gl}+\delta_g) - C(h_{gl})] + \mu\,[C(h_{bl}-\delta_b) - C(h_{bl})] \\ &= (1-\mu)pC'(h_{gl})\delta_g - \mu C'(h_{bl})\delta_b = \mu\delta_b\left[C'(h_{gl}) - C'(h_{bl})\right] < 0,\end{aligned}$$

where the last inequality follows from strict convexity of resource costs with $h_{gl} < h_{bl}$, implying that h_0 cannot be a cost-efficient allocation. Thus, a cost-efficient allocation is characterized by binding incentive constraints of bad-signal signals and $h_{bl} = h_{gl} = h_l$. Using this, Step 1 and resource feasibility imply $h_l > u(y-l)$ and $h_{gy} < u(y)$. Finally, the incentive constraint of the good-signal agents can only be met with equality if $c_{gy} = 0$ now that we have established $h_{gl} = h_{bl}$. However, this can not be optimal because $u(c)$ satisfies Inada conditions, $\lim_{c \mapsto 0} u'(c) = \infty$ and hence an infinitely small redistribution back from c_l to c_{gy} yields infinite improvements in lifetime utility, increasing profits. Thus, setting $c_{gy} = 0$ can not be optimal and the good-signal agent incentive compatibility constraint must be slack. ∎

So far we established that only incentive constraints of good-signal agents are slack as well as individual rationality constraints of bad-signal agents. Further, we have shown that $h_l > u(y-l)$ and $h_{gy} < u(y)$. In the remaining two steps, we demonstrate first that individual rationality constraints of good-signal agents are binding, completing the proof of Lemma 1, Part (ii), and afterwards that $h_{gy} \geq h_l$ at the cost-efficient allocation, completing the proof of Lemma 1, Part (iii).

Step 3 *Let h be a cost-efficient allocation with trade. Then the individual rationality constraint* *of the good-signal agents are binding.*

Proof. Suppose not, such that there is not only slack $\varepsilon_b^{ir} > 0$ but also a slack $\varepsilon_g^{ir} > 0$ in the good-signal agents individual rationality constraint (6). Let $h_0 = \{h_l, h_{gy}\}$ be the cost-efficient allocation with trade in that case. This cannot be the cost-efficient allocation because further resources can be saved by choosing an allocation $h_1 = \{h_l, h_{gy} - \delta\}$ as long as

$$\delta \leq \min\left\{\frac{\varepsilon_b}{\beta(1-\beta^N)(1-\mu)(1-p)}, \frac{\varepsilon_g}{1-\beta+\beta(1-\beta^N)(1-\mu)(1-p)}\right\},$$

which ensures that individual rationality is satisfied for good and bad-signal agents. Choosing such allocation decreases the resource costs by $(1-\mu)(1-p)\left[C(h_{gy}) - C(h_{gy}-\delta)\right]$, contradicting that the good-signal individual rationality constraints are not binding at the cost-efficient allocation. ■

Step 4 *Let h be a cost-efficient allocation with trade. Then, $h_{gy} \geq h_l$.*

Proof. Suppose not, and without loss of generality, let $h_l = h_{gy} + \varepsilon$ for $h_0 = \{h_l, h_{gy}\}$ with ε arbitrarily small. Then, an insurer can improve by deviating to $h_1 = \{h_{bl} = h_l, h_{gl} = h_l - \delta_{gl}, h_{gy} + \delta_{gy}\}$ with $\delta_{gl} = \frac{1-p}{p}\delta_{gy}$, and $\delta_{gy} \leq \frac{\varepsilon}{2}$. This perturbation doesn't affect the good-signal individual rationality constraint, but by convexity of resource costs it marginally increases the insurer's profits by $(1-\mu)(1-p)\left[C'(h_{gy}+\varepsilon) - C'(h_{gy})\right]\delta_{gy}$. ■

B.2 Proof of Proposition 1

To see why autarky cannot be a cost-efficient allocation, consider an insurance scheme that involves only transfers between good-signal agents, so that $h_{gy} = u(y-\delta)$ and $h_{gl} = u(y-l+\gamma_g)$, $\delta, \gamma_g > 0$, in line with resource feasibility:

$$(1-p)\delta \geq p\gamma_g.$$

The scheme provides insurance, leads to higher utility by concavity, to lower resource costs by convexity, and therefore at least weakly dominates autarky in terms of resources saved. What remains to be shown is that the scheme is also implementable. First, such a scheme affects the individual rationality constraints of good-signal agents as follows

$$\begin{aligned}
&(1-\beta)\left[pu'(y-l)\gamma_g - (1-p)u'(y)\delta\right] + \beta(1-\beta^N)(1-\mu)\left[pu'(y-l)\gamma_g - (1-p)u'(y)\delta\right] \\
&= \left[u'(y-l) - u'(y)\right]\left[(1-\beta) + \beta(1-\beta^N)(1-\mu)\right]p\gamma_g \\
&\geq 0.
\end{aligned}$$

The second line uses resource feasibility, the third one that $\left[(1-\beta)+\beta(1-\beta^N)(1-\mu)\right]$, p, γ_g are strictly positive; The positive sign then follows for $l > 0$, and strict concavity of utility. Per construction, such a scheme is consistent with resource feasibility and also satisfies individual rationality constraints of bad-signal agents who remain at autarky. Thus, autarky cannot be a cost-efficient allocation.

B.3 Random return from autarky

An alternative possibility to model exclusion from insurance comes from the literature of sovereign default. With probability $0 \leq \theta \leq 1$, agents are offered an insurance contract again after defaulting as in Eaton and Gersovitz (1981) and Arellano (2008). With a constant probability to return each period, the average number of total exclusion periods (including teh current period) is $\mathrm{E}(N) = 1/\theta$. Incentive constraints and resource feasibility are unaffected by this change, and so is the definitions of lifetime utility $\tilde{w}(h)$. The value of autarky U^{Aut} now satisfies the following recursive equation:

$$U^{Aut} = (1-\beta)\left\{\mu u(y-l) + (1-\mu)\left[pu(y-l) + (1-p)u(y)\right]\right\} + \beta\theta\tilde{w} + \beta(1-\theta)U^{Aut} \quad (24)$$

Individual rationality constraints of good and bad-signal agents now read

$$\underbrace{(1-\beta)\left[pu\left(y-l+\gamma_g\right) + (1-p)u\left(y-\delta\right)\right] + \beta(1-\theta)\tilde{w}}_{\tilde{w}(g)} \geq \underbrace{(1-\beta)\left[pu(y-l) + (1-p)u(y)\right] + \beta(1-\theta)U^{Aut}}_{U^{Aut}(g)}, \quad (25)$$

and

$$\underbrace{(1-\beta)u(y-l+\gamma) + \beta(1-\theta)\tilde{w}}_{\tilde{w}(b)} \geq \underbrace{(1-\beta)u(y-l) + \beta(1-\theta)U^{Aut}}_{U^{Aut}(b)}. \quad (26)$$

In this economy, the no-trade condition is as follows.

Theorem 2 (No Trade) *The autarky allocation $\{y, y-l, y-l\}$ is the only implementable allocation if and only if*

$$\frac{u'\left(y-l\right)}{u'\left(y\right)} \leq \underbrace{T_s(p,\mu)T_r(p,\mu,\beta,\theta)}_{T_d(p,\mu,\beta,\theta)}$$

with $T_s(p,\mu)$ as the single-interaction pooled price ratio

$$T_s(p,\mu) = \frac{\mu + (1-\mu)\,p}{(1-\mu)\,p} = \frac{\mathbb{E}\left[P|P \geq p\right]}{1-\mathbb{E}\left[P|P \geq p\right]}\frac{1-p}{p}$$

and with $T_r(p,\mu,\beta,\theta)$ as a diminishing factor resulting from repeated interactions

$$T_r(p,\mu,\beta,\theta) = \frac{p\left[1-(1-\theta)\beta\mu\right]}{p+(1-p)\beta(1-\theta)\mu}, \quad 0 < T_r \leq 1$$

and $0 < T_d(p,\mu,\beta,\theta) \leq T_s(p,\mu)$ as the repeated-interactions pooled price ratio for $0 \leq \theta \leq 1$.

Proof. The proof follows analogous steps as the one of Theorem 1.

Sufficiency We arrive at the following constraint on the FOC of the profit maximisation problem of an insurer (the right hand side equality):

$$\frac{u'\left(y-l\right)}{u'\left(y\right)} \leq \frac{\mu+(1-\mu)\,p}{(1-\mu)\,p}\frac{p\left[1-\beta(1-\theta)\mu\right]}{p\left[1-\beta(1-\theta)\mu\right]+\beta(1-\theta)\mu} = \frac{u'\left(c_l\right)}{u'\left(c_{gy}\right)},$$

Imposing the no-trade condition (the left hand side inequality) implies this can only be satisfied by $c_l = y - l$ and $c_{gy} = y$.

Necessity As in the proof of Theorem 1, we consider an allocation with trade, $h = \{h_{gy} = u(y-\delta), h_l = u(y-l+\gamma)\}$, which, if implementable, yields the following for the individual rationality constraints:

$$\tilde{w}(g) = U^{Aut}(g) \tag{27}$$

$$\tilde{w}(b) - U^{Aut}(b) = \epsilon > 0. \tag{28}$$

Following similar steps as in the proof of Theorem 1, the slack ϵ in the bad-signal agents individual rationality constraint of bad-signal agents is

$$\epsilon = \frac{(1-\beta)}{1-\beta(1-\theta)\mu} u'(y-l)\gamma. \tag{29}$$

We get the counterpart to (23) from the binding individual rationality constraints of the good-signal agents:

$$\begin{aligned} &0 = \tilde{w}(g) - U^{Aut}(g) = (1-\beta)\left[-(1-p)u'(y)\delta + pu'(y-l)\gamma\right] + \beta(1-\theta)\mu\epsilon \implies \\ &\frac{(1-\mu)(1-p)\delta}{[\mu+(1-\mu)p]\gamma} = \frac{u'(y-l)}{u'(y)} \frac{(1-\mu)p}{\mu+(1-\mu)p} \frac{p[1-\beta(1-\theta)\mu]}{p[1-\beta(1-\theta)\mu]+\beta(1-\theta)\mu}. \end{aligned} \tag{30}$$

Assuming that the no-trade condition is violated yields

$$\frac{u'(y-l)}{u'(y)} > \frac{\mu+(1-\mu)p}{(1-\mu)p} \frac{p[1-\beta(1-\theta)\mu]}{p[1-\beta(1-\theta)\mu]+\beta(1-\theta)\mu},$$

combining with (30) implies that the allocation h generates positive resource savings. Hence, autarky cannot be the cost-efficient allocation, concluding the proof. ■

Equivalence between deterministic and random exclusion Comparing Theorem 1 and Theorem 2, we can establish equivalence between the two formulations if the diminishing factors $T_r(p,\mu,\beta,\theta)$ and $T_r(p,\mu,\beta,N)$ are equal. The two factors for fixed-length and random-length exclusion are

$$T_r(p,\mu,\beta,N) = \left\{ \frac{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p}{\left[1-\beta\mu-\beta^{N+1}(1-\mu)\right]p + \beta(1-\beta^N)\mu} \right\}$$

and

$$T_r(p,\mu,\beta,\theta) = \frac{p[1-\beta(1-\theta)\mu]}{p[1-\beta(1-\theta)\mu]+\beta(1-\theta)\mu}.$$

The equivalence between the two is that $T_r(p,\mu,\beta,N=0) = T_r(p,\mu,\beta,\theta=1) = 1$ and $\lim_{N\mapsto\infty} T_r(p,\mu,\beta,N) = T_r(p,\mu,\beta,\theta=0) = \frac{p(1-\beta\mu)}{p(1-\beta\mu)+\beta\mu}$. The explicit relationship between N and θ is:

$$\theta = \frac{\beta^N(1-\beta)}{1-\beta^{N+1}},$$

which is confirmed by direct evaluation of the two no-trade conditions.

B.4 A life-cycle economy

Agents live for K periods. Each period a new generation is born.[7] All generations are of equal measure set to $\frac{1}{K}$ so that the total measure of all agents in the economy is equal to 1. In period t the age of a generation who entered in $t-k$ is k and we will use k as age index. We consider a problem of a monopolist insurer who can exclude agents from the contract if they default. The insurer offers memoryless contracts which can be conditioned on agent's age, leading to utility allocations $h = \left\{h^k_{bl}, h^k_{gl}, h^k_{gy}\right\}_{k=1}^{K}$.

For memoryless contracts, the future utilities don't depend on the current private signal hence the incentive compatibility constraints are similar to those in the paper:

$$h^k_{bl} \geq h^k_{gl},$$
$$ph^k_{gl} + (1-p)h^k_{gy} \geq ph^k_{bl} + (1-p)u(0),$$

and hence due to the incentive compatibility constraint of the bad signal agents binding[8] we constrain attention to contracts specifying utility allocations $h = \left\{h^k_{l}, h^k_{gy}\right\}_{k=1}^{K}$. Every contract implies a sequence of expected k-period utilities:

$$\tilde{w}^k = (1-\beta)\left[(\mu + (1-\mu)p)\, h^k_l + (1-\mu)(1-p)h^k_{gy}\right],$$

and the expected utility in autarky in every period is:

$$\tilde{U}^{Aut} = (1-\beta)\left[(\mu + (1-\mu)p)\, u(y-l) + (1-\mu)(1-p)u(y)\right].$$

Deterministic exclusion With exclusion of $N \geq 1$ future periods and $k < K$, the individual rationality constraints read:[9]

$$(1-\beta)h^k_l + \sum_{t=k+1}^{\min\{k+N,K\}} \beta^{t-k}\tilde{w}^t \geq (1-\beta)u(y-l) + \sum_{t=k+1}^{\min\{k+N,K\}} \beta^{t-k}\tilde{U}^{Aut}$$

$$(1-\beta)\left[ph^k_l + (1-p)h^k_{gy}\right] + \sum_{t=k+1}^{\min\{k+N,K\}} \beta^{t-k}\tilde{w}^t \geq (1-\beta)\left[pu(y-l) + (1-p)u(y)\right] + \sum_{t=k+1}^{\min\{k+N,K\}} \beta^{t-k}\tilde{U}^{Aut}$$

[7] An alternative possibility is to assume the insurer can transfer resources between periods.

[8] This follows from a standard backward induction argument.

[9] Note, here we assume N to be an integer. The formulae can be adapted to have $N \in \mathbb{R}_{\geq 0}$, for example, by taking an appropriately weighted average of the expected utility of the contract and the outside option in the last non-integer part of the exclusion period. Say, if $\lceil N \rceil > N > \lfloor N \rfloor$ then let $m = N - \lfloor N \rfloor$ and for the last part of the exclusion period add $m\tilde{w}^{k+\lceil N \rceil}$.

When either $N = 0$ or $k = K$, these constraints reduce to comparing instantaneous utilities only:

$$(1-\beta)h_l^k \geq (1-\beta)u(y-l), \tag{31}$$

$$(1-\beta)\left[ph_l^k + (1-p)h_{gy}^k\right] \geq (1-\beta)\left[pu(y-l) + (1-p)u(y)\right]. \tag{32}$$

Thus, the set of individual rationality constraints is qualitatively similar when there is no exclusion to the case of the oldest generation agents and contracts with exclusion. The objective of the insurer is to maximize the slack on the resource constraint:

$$\max_{\{h_l^k, h_{gy}^k\}_{k=1}^K} \quad [\mu + (1-\mu)p]\, C(u(y-l)) + (1-\mu)(1-p)C(u(y))$$
$$-\frac{1}{K}\sum_{k=1}^{K}\left[[\mu + (1-\mu)p]\, C(h_l^k) + (1-\mu)(1-p)C(h_{gy}^k)\right]$$

Random exclusion The incentive compatibility and the resource constraint slack functions are unchanged when agents can randomly return from autarky. For the individual rationality constraints, it's convenient to define the expected remaining lifetime utilities of the contract and autarky, as follows:

$$\hat{w}^k = \begin{cases} (1-\beta)\left[((1-\mu)\,p+\mu)\,h_l^k + (1-\mu)\,(1-p)\,h_{gy}^k\right] & \text{if } k = K \\ (1-\beta)\left[((1-\mu)\,p+\mu)\,h_l^k + (1-\mu)\,(1-p)\,h_{gy}^k\right] + \beta\hat{w}^{k+1} & \text{for } 1 \leq k < K \end{cases}$$

These can be used to define corresponding objects for autarky:

$$\hat{U}^{Aut,k} = \begin{cases} (1-\beta)\left[((1-\mu)\,p+\mu)\,u(y-l) + (1-\mu)\,(1-p)\,u(y)\right] \text{ if } \quad k = K \\ (1-\beta)\left[((1-\mu)\,p+\mu)\,u(y-l) + (1-\mu)\,(1-p)\,u(y)\right] \\ +\beta\left(\theta\hat{w}^{k+1} + (1-\theta)\hat{U}^{Aut,k+1}\right) \text{ for } 1 \leq k < K. \end{cases}$$

The individual rationality constraints for $k < K$ are:

$$(1-\beta)h_l^k + \beta(1-\theta)\hat{w}^{k+1} \geq (1-\beta)u(y-l) + \beta(1-\theta)\hat{U}^{Aut,k+1}$$
$$(1-\beta)\left(ph_l^k + (1-p)h_{gy}^k\right) + \beta(1-\theta)\hat{w}^{k+1} \geq$$
$$(1-\beta)\left(pu(y-l) + (1-p)u(y)\right) + \beta(1-\theta)\hat{U}^{Aut,k+1}$$

We then calculate the cost-efficient contracts and plot the resources saved by the insurer against N and θ on Figure 1. To do this, we set $y = 1$ and then compute l to yield the ratio of marginal utilities reported as the willingness to pay in Table 1, taking the values of β, p and μ from Table 1.

If exclusion is not possible, that is, when either $N = 0$ or $\theta = 1$, the insurer can not improve upon the no-trade allocation and makes zero profits. However, we find that for $N \geq 1$ the insurer

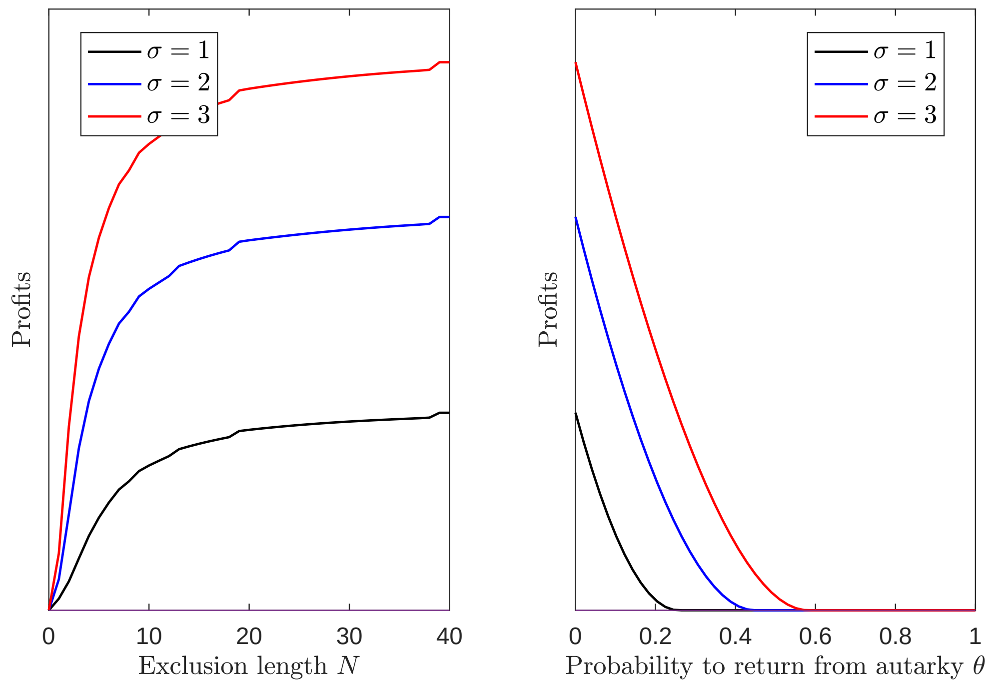

Figure 1: Insurer profits (as share in total resources) for different exclusion length (deterministic - left panel, random - right panel) and risk aversion.

already incurs positive profits in case of deterministic exclusion for all three risk-aversion values. For random return specification, the threshold values for θ are 0.27, 0.45 and 0.57, for $\sigma = 1, 2, 3$ respectively. These imply the expected length of future exclusion $\frac{1}{\theta} - 1$ of 2.7, 1.22 and 0.75 years, respectively.