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ARTYKUŁY

Maciej K. Dudek*

ON EFFICIENT TAXATION

1. INTRODUCTION

Taxes at a very fundamental level allow governments to collect revenue, but, typically, create distortions that lead to welfare losses larger than those implied by the tax liabilities themselves. This basic observation leads to a natural, but yet profound, economic problem. How should a policy maker with given revenue needs R decide on the proper form of tax function? This question can be, and has been, answered at many levels. In this paper we propose a novel and at the same time very simple approach to the issue and identify tax functions that solve the optimization problem of the policy maker.

The issue analyzed in the paper is not original. Many researchers have tried to characterize the desired tax function. In fact, extensive and successful literature examines the concept of optimal taxation. Specifically, the shape of the optimal tax function has been of interest to economists for a number of decades now. The contribution of J.A. Mirrlees (1971) offered enough structure on the optimal taxation problem to allow for the derivation of the relevant conditions that define the optimal tax function. Despite the complexity of the efficiency conditions, researchers have been able to resort to numerical techniques to identify the shapes of tax functions that solve the optimal taxation problem as defined by Mirrlees. Furthermore, more recent and ingenious approaches of P.A. Diamond (1998) and E. Saez (2001) have allowed to relate the underlying optimal taxation problem to observables and generate policy relevant implications.

In this paper, we choose to embark on a path that parallels that typically explored in the literature. Specifically, we focus on tax functions that, in addition

* Vistula University and INE PAN. The author bears complete responsibility for the content of this paper. No institution should be held responsible for errors that remain. Please send your comments to mkdudek@alumni.caltech.edu.

to ensuring that the revenue needs are met, lead to the lowest possible level of distortions in the economy, i.e., to tax functions, termed *efficient*, that ensure the highest possible degree of efficiency in the economy. Our approach, thus, is much more modest than, but at the same time can be considered dual to, that normally studied in the literature. We are for the most part interested in efficiency whereas the main stream of the literature takes a broader perspective and focuses on welfare.

At the technical level, we choose, following Saez (1999), to identify departures from efficiency with dead weight burden. Furthermore, initially, we choose to abstract completely from any equity considerations and focus strictly on efficiency issues. Specifically, we are interested in identifying tax functions that minimize the dead weight burden imposed on the society. The problem of minimizing the dead weight burden has not been subject to extensive research with the exception of the work of Saez (1999), who derives an explicit analytic expression that defines the dead weight burden minimization function when lump sum taxes are unavailable and individual productivities are not observable. It appears that the literature has mostly ignored the problem of dead weight burden minimization as the problem in its purest form is trivial, and the solution can be degenerate, and, most importantly, the problem has been superseded by a more general problem of optimal taxation. In this paper, we resurrect the problem of dead weight burden minimization and argue that the problem can be of practical relevance. Furthermore, we show that the tax function that ensures efficiency is of particularly simple form and that it is, contrary to the findings of Saez (1999), locally flat implying mostly zero marginal tax rates. In addition, we extend our analysis and, following J.A. Mirrlees (1971), allow for aversion towards inequality on the part of policy makers. We prove that our underlying result is robust, i.e., the shape of the efficient tax function does not change in the qualitative sense, i.e., it remains locally flat, if one introduces a motive for redistribution.

It may appear unnecessary to even examine the issue of efficient taxation given the success of the optimal taxation program.We do not share this view as the problem of efficient taxation can be of interest on its own for a number of reasons. First of all, the problem has not been subject to extensive research apart from the effort by Saez (1999). Moreover, as we argue in the paper the approach to the problem proposed by Saez (1999), despite being mathematically sound, is too restrictive and fails to identify the tax function of interest properly. Consequently, it appears worthwhile to recast the problem and identify the true efficient tax function.

Secondly, the problem of efficient taxation – studied in this paper – is more natural and better grounded in economic theory than the canonical problem of optimal taxation. Observe that relying in the process of evaluating different tax function on the dead weight burden rather than on the levels of utility makes actual comparisons meaningful, and by doing so removes the most critical objection to the theory of optimal taxation. Formally, in our approach, a given tax function is evaluated using a well-defined metric expressed in observable units and consistent with individual preferences rather than being judged on a metric based on arbitrary units deprived, according to elementary economic theory, of any economic meaning.

Thirdly, following Mirrlees we allow policy makers to exhibit aversion toward inequality. Specifically, we assume that the preferences of policy makers are defined directly over the magnitudes of dead weight burden borne by individual economic agents. Naturally, such an approach does not suffer from the shortcoming of the standard approach that relies on the notion, occasionally criticized see J.E. Stiglitz (1987), that individual utilities can be subject to cross personal comparisons. Clearly, by defining the preferences directly over meaningful and comparable concepts we not only extend the literature, but also present a framework that can be seen as deeply grounded in economic theory rather than being contrived to deal with a problem at hand.

We establish our findings in a very simple manner that involves only elementary calculus and very rudimentary distortions of tax functions. Specifically, our technique is very simple and is based on very basic lump-sum distortions coupled with a replacement of a part of tax function with a flat portion. We show that distortions of this form, when applied to the efficient tax function, can be revenue neutral while reducing at the same time the amount of distortions in the economy. Consequently, we argue that a given tax function cannot be efficient unless it is locally flat. Furthermore, at the technical level, we use, following Saez (2001) and Diamond (1998), a utility function that is essentially of a quasi-linear form. Such a specification is not innocuous and makes our underlying method applicable. Nevertheless, our specific choice of utility function does not, as pointed out by Saez (2001), contradict empirical evidence as it appears that actual income effects are small.

The approach to optimal taxation proposed by Mirrlees has rightly gained prominence in the literature. Consequently, many readers may consider modifications of the original problem, even those offering new insights and as natural as ours, to be unnecessary. Furthermore, numerous researchers may believe that our assumptions essentially allow us, especially given the simplicity of our approach, to trivialize a serious economic problem. We strongly disagree, as we study a legitimate problem, and obtain our results in a framework that is frequently analyzed in the literature, e.g., Diamond (1998). Furthermore, many researchers including Saez and S. Stantcheva (2012), and J. Werning (2007), have pointed out that the original approach of Mirrlees may be too restrictive. Consequently, they argue, it may be necessary to reformulate the original optimal taxation problem to allow for a richer class of objective functions than the utilitarian criterion – a challenge we accept in this paper. Moreover, our results, at a technical level, do not stand in conflict with the literature on optimal taxation. We in fact consider our findings to be complementary to the results of Diamond (1998), Saez (2001), and Mirrlees (1971), as we effectively enrich the class of problems and the class of solutions to taxation problems.

This paper is divided into five sections. We outline the basic problem in the next section. In the following section we present the solution to a simple problem of dead weight burden minimization. In section four we extend our approach and allow for the presence of a motive for redistribution. Finally, section five contains conclusions.

2. BASIC PROBLEM

Following Saez (2001) and effectively Diamond (1998), let us consider an economy populated with economic agents whose preferences are represented with the following utility function

$$U(c, L) = h\left(c - \frac{1}{2}L^2\right), \tag{1}$$

where $h(|)$ is an increasing and differentiable function. For the most part, it can be assumed, following Saez (2001) and effectively by Diamond (1998) , that $h(|)$ is simply a logarithmic function. Note that the utility function is effectively of a quasi-linear form, which is, as noted by Saez (2001), consistent with observables. Our assumption with regard to the form of the utility function is not innocuous as the functional form makes our underlying method applicable. Furthermore, at the technical level the presence of $h(|)$ in specification (1) ensures that some unrealistic tax function – those leading to tax liabilities higher than income – are automatically excluded.

Furthermore, let us assume that $[a_L, |aH]$ represents the support of the relevant distribution of skills and let $f(|)$ denote the corresponding *pdf* of the distribution of skills. Naturally, we assume that an agent whose productivity is equal to a delivers $y = aL$ units of output when she chooses to supply L units of labor.

The problem of an economic agent whose productivity is equal to a and who faces tax function $\tau(y)$ can be summarized as

$$\max_{\{y\}} U\left(c = y - \tau(y), L = \frac{y}{a}\right) = h\left(y - \tau(y) - \frac{1}{2}\left(\frac{y}{a}\right)^2\right), \tag{2}$$

in which we treat consumption as the numeraire.

The relevant first order condition can be expressed as

$$h'\left(y - \tau(y) - \frac{1}{2}\left(\frac{y}{a}\right)^2\right)\left(1 - \tau'(y) - \frac{y}{a^2}\right) = 0, \tag{3}$$

which implicitly defines the optimal[1] income earned, y.

Let us denote the optimal choice of an agent whose productivity is equal to a with y_a. Furthermore, let R_a denote the revenue collected from an agent whose productivity is equal to a, i.e., let $Ra | = | \tau(y_a)$. Finally, let $U_a = h\left(y_a - \tau(y_a) - \frac{1}{2}\left(\frac{y_a}{a}\right)^2\right)$.

[1] Occasionally the first order condition is not sufficient, a fact relevant in the context of optimal taxation, as pointed out by S. Lollivier and J.-Ch.Rochet (1983), and by U. Ebert (1992) .

denote the realized utility of an agent whose productivity is equal to a.

Before we proceed further we want to acknowledge that in this paper we restrict attention to tax functions $\tau(|)$ that satisfy

$$\forall y| \in |R\,|+| \cup \{0\}|\, \tau(y)\,| \leqslant |\, y, \tag{4}$$

i.e., to tax functions that are feasible.

Furthermore, in the main part of the paper, we assume[2] that $0 < a_L$, and that $a_H < \infty$. Note that in this case the revenue collected given tax function $\tau(|)$ is given by

$$R = \int_{a_L}^{a_H} R_a f(a) da. \tag{5}$$

Moreover, we assume, to exclude the possibility of trivial solutions, that R is big enough, so that a uniform tax of $\dfrac{R}{\int_{a_L}^{a_H} f(a) da}$ on all agents is not feasible, i.e., we assume that

$$R = \int_{a_L}^{a_H} R_a f(a) da. \tag{6}$$

At this stage we choose to focus on the case when all agents supply a strictly positive amount of labor given $\tau(|)$. Specifically, we assume that

$$\forall a \in [a_L, a_H] \Big| \, h(-\tau(0)) < h\left(y_a - \tau(y_a) - \frac{1}{2}\left(\frac{y_a}{a}\right)^2\right), \tag{7}$$

where y_a satisfies restriction (3). We consider the alternative case, when some agents choose to be idle in the online Appendix E.

The concept of dead weight burden is routinely used in applied work. Nevertheless, there is disagreement with regard to its precise meaning. In this paper, we follow Saez (1999) and focus on the definition, which appears to be most commonly accepted in the literature. Specifically, we choose to measure the dead weight burden relying on the notion of equivalent variation, i.e., we define the dead weight burden as the negative of the difference between taxes actually paid by an individual and the lump sum tax that would induce the same level of utility.

The tax function, $\tau(|)$, is of general form and presumably creates some distortions. Therefore, collecting the specific amount of revenue, $\tau(y_a)$, comes at a cost to the consumer that goes beyond the direct revenue cost. The concept of dead weight burden that we employ here is related to that additional cost. In fact, typically, the consumer could be willing to pay more in taxes to avoid this additional cost. Specifically, if lump sum taxes are available, then there are no distortions, assuming that a is large enough, and the level of income earned by a consumer whose productivity is equal to a is given by

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[2] We discuss the case when $a_H = \infty$ in the online Appendix D.

$$y_a^F = a^2, \tag{8}$$

which leads to the value of realized utility of

$$U_a^F = h\left(\frac{1}{2}a^2 - T_a\right), \tag{9}$$

where T_a denotes the lump sum tax actually paid by the agent.

We can now find the value of T_a, which makes the consumer just indifferent to facing the original tax function, $\tau(\,|\,)$, and paying T_a by equating U_a and U_a^F, which yields

$$h\left(y_a - \tau(y_a) - \frac{1}{2}\left(\frac{y_a}{a}\right)^2\right) = h\left(\frac{1}{2}a^2 - T_a\right) \tag{10}$$

and further reduces to

$$T_a = \frac{1}{2}\left(\frac{a^2 - y_a}{a}\right)^2 + \tau(y_a). \tag{11}$$

Observe that originally the agent actually paid $\tau(y_a)$ in taxes and, at the same time, could be willing to pay up to T_a, given by (11), if lump sum taxes were available. Alternatively, we can say that the discrepancy between T_a and $\tau(y_a)$ reflects the additional cost born by the agent due to the presence of the distortions induced by $\tau(\,|\,)$. We choose to identify this *unnecessary* revenue loss stemming from existing distortions with the dead weight burden. Consequently, we have

$$DWB_a = T_a - \tau(y_a) = \frac{1}{2}\left(\frac{a^2 - y_a}{a}\right)^2. \tag{12}$$

The level of distortion quantified with expression (12) captures the unnecessary burden borne by economic agents when taxes are distortionary. In our analysis we assume that minimizing this unnecessary burden while ensuring that proper revenue needs are met is the principal motive of the government policy.

3. DEAD WEIGHT BURDEN

We are in a position to define the problem of the government. First of all, we assume that individual characteristics are not publicly observable. Furthermore, we assume that the government is interested in minimizing the economy-wide dead weight burden. Therefore, we can state the objective function as

$$\min_{\{\tau(\;)\}} DWB = \int_{a_L}^{a_H} DWB_a f(a)da. \tag{13}$$

Note that individual dead weight burdens are expressed in common units. Therefore, we do not encounter standard comparability issues by choosing to express the objective function with equation (13).

Naturally, the government must meet its specific revenue needs, R, and must take into account the fact that economic agents rationally respond to the tax schedule. Formally we state the problem as

$$\min_{\{\tau(\ )\}} DWB = \int_{a_L}^{a_H} \frac{1}{2}\left(\frac{a^2 - y_a}{a}\right)^2 f(a)da. \tag{14}$$

subject to

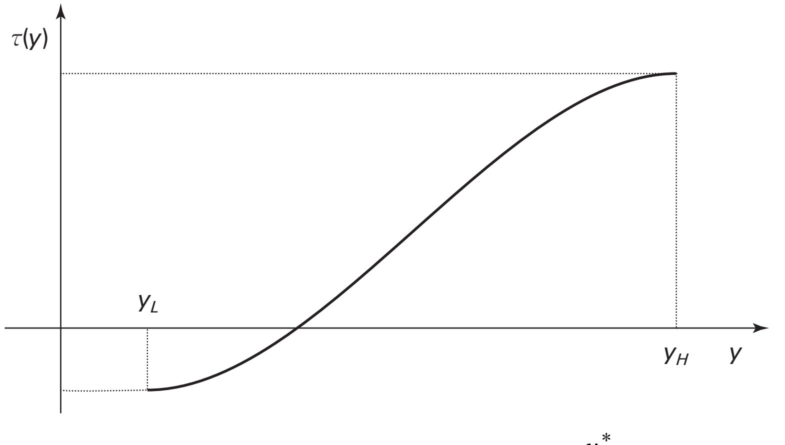
$$R = \int_{a_L}^{a_H} R_a f(a)da \tag{15}$$

and to

$$h'\left(y - \tau(y) - \frac{1}{2}\left(\frac{y}{a}\right)^2\right)\left(1 - \tau'(y) - \frac{y}{a^2}\right) = 0. \tag{16}$$

The above optimization problem, expressed with equations (14), (15), and (16), can be dealt with using the proper mathematical techniques. Let $\tau_E(|)$ be the solution, possibly obtained with the approach of Saez (1999) applied to bounded distributions, of the above problem. Naturally, at this stage we cannot state anything about the qualitative features of $\tau_E(|)$. However, for purely illustrative purposes let us assume that the actual shape resembles that depicted in figure (1) of the optimal tax function described by E. Sadka (1976), and Seade (1977, 1982).

Figure 1. The Generic Form of an Optimal Tax Function



Let y_a^* denote the level of income earned and $L_a^* = \dfrac{y_a^*}{a}$ the corresponding choice of labor supply of a consumer whose productivity is equal to a and who faces the tax schedule, $\tau_E(|)$. Observe that the revenue brought by an agent whose productivity is equal to a is given by $R_a^* = \tau_E(y_a^*)$ and that the total amount of revenue collected must be equal to R, i.e.,

$$R = \int_{a_L}^{a_H} \tau(y_a^*) f(a) da. \tag{17}$$

Furthermore, the realized value of the objective functional is given by

$$DWB^* = \int_{a_L}^{a_H} \frac{1}{2}\left(\frac{a^2 - y_a^*}{a}\right)^2 f(a) da, \tag{18}$$

and the realized utility of an individual whose productivity is equal to a can be written as

$$U_a^* = h\left(y_a^* - \tau(y_a^*) - \frac{1}{2}\left(\frac{y_a^*}{a}\right)^2\right). \tag{19}$$

Recall that we have already assumed that all agents prefer to supply a strictly positive amount of labor, the alternative case is considered in the online Appendix E, when they face $\tau_E(|)$, i.e., that

$$\forall a \in [a_L, a_H] \| y_a^* > 0. \tag{20}$$

Let Y^C denote the set of income levels actually chosen by someone, i.e., let

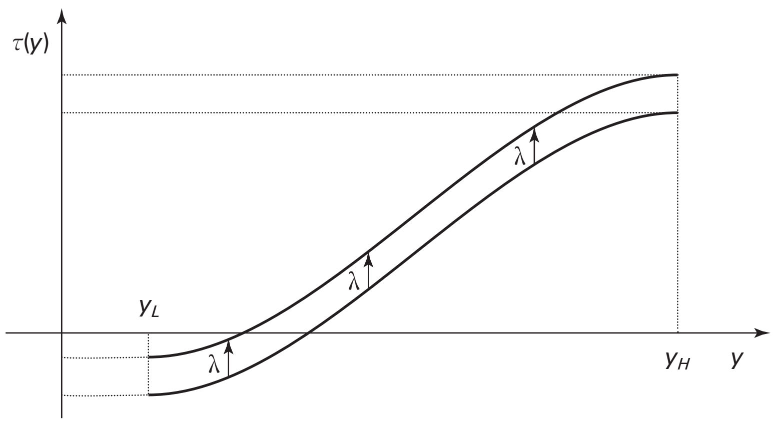
$$Y^C = \{y_a^* | a \in [a_L, a_H]\}. \tag{21}$$

Furthermore, let us now consider a simple variation of $\tau_E(y)$. Specifically, let us consider the following tax function

$$\tau_\lambda(y) = \begin{cases} y & \text{if } y \notin Y^C \\ \tau_E(y) & \text{if } y \in Y^C \end{cases} \tag{22}$$

where λ is a, possibly very small, constant. Figure (2) presents the new tax function, $\tau_\lambda(|)$.

Figure 2. The form of $\tau_\lambda(|)$ obtained from $\tau_E(|)$ with an upward shift by λ over the range of actually chosen *y*



Note that the new tax function given by (22) is simply equal to the previous one shifted up by λ for the relevant range of y. Observe that given our assumptions of strictly positive labor supply on the part of all agents, restriction (7), it must be the case, given the form of the utility function, that the labor supply choice of agents will remain unchanged when tax liabilities are dictated with $\tau_{\lambda}(|)$ rather than by $\tau_E(|)$for λ sufficiently small[3]. Furthermore, given the form of the utility function and that λ is a lump sum transfer, there should be no further efficiency losses as compared to the case when tax liabilities are determined with $\tau_E(|)$. However, the values of individual utilities are affected, and they now become

$$U_a^*(\lambda) = h\left(y_a^* - \tau(y_a^*) - \lambda - \frac{1}{2}\left(\frac{y_a^*}{a}\right)^2\right). \tag{23}$$

Similarly, we can now express the revenue collected from a single individual as

$$R_a^*(\lambda) = R_a^* + \lambda = \tau_E(y_a^*) + \lambda, \tag{24}$$

which, in particular, implies that the total amount of revenue collected by the government is higher and given by

$$R(\lambda) = \int_{a_L}^{a_H} R_a(\lambda) f(a)da = R + \lambda \int_{a_L}^{a_H} f(a)da, \tag{25}$$

i.e., the government collects more revenue than it needs.

Let us now consider the following class of tax functions

$$\tau_{\overline{y}}(y) = \begin{cases} \tau_{\lambda}(y) \ for \ y < \overline{y} \\ \tau_{\lambda}(\overline{y}) \ for \ \overline{y} < y. \end{cases} \tag{26}$$

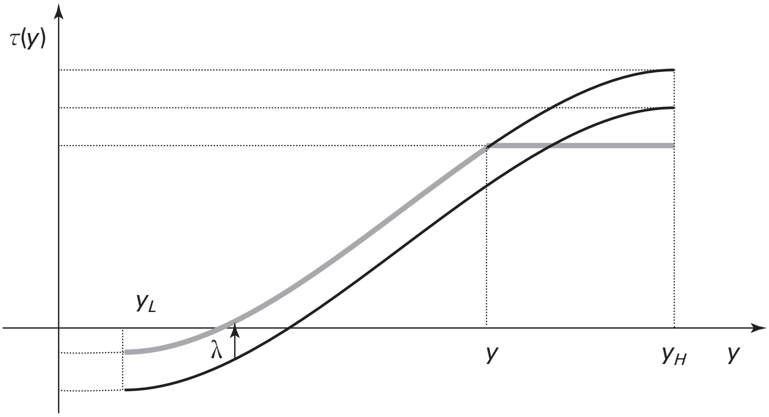
Naturally, $\tau_y(|)$ looks just like $\tau_{\lambda}(|)$, with its right hand end flattened starting from $\overline{y}$, figure (3).

Note that such a modification lowers the marginal tax rate for high income earners and it lowers the actual tax liabilities of high income earners. If anything such a modification reduces the relative burden on taxation imposed on high income earners. Moreover, as compared to the starting point the actual tax liabilities of low income earners are now higher and the tax liabilities of the high income earners are now lower, i.e., we deal with a reduction in the overall redistribution of income.

Let us denote with y_L the level of income earned by the lowest type when she faces $\tau_E(|)$ and y_H denotes the level of income earned by the highest type when taxes are paid according to $\tau_E(|)$ Furthermore, let us assume that all values of

[3] Note that $\tau_E(|)$s shifted up by λ for all values of y that were originally chosen. Furthermore, for λ small enough it must the case that all agents remain active as originally by assumption they *strictly* preferred to be active.

Figure 3. The Form of $\tau_{\bar{y}R}(|)$



income $y \;|\in|\; [yL, \;|y_H]$ are actually chosen by someone[4]. Imagine, now, that tax liabilities are to be dictated by $\tau_y(|)$ rather than by $\tau_E(|)$ Observe that when we choose $\bar{y} = y_L$ then the revenue collected from all agents must be, for λ small enough, necessarily smaller than the one that is collected from all agents when taxes are paid in line with $\tau_E(|)$ Furthermore, when $\bar{y} = y_H$ then $\tau_y(|) = \tau_\lambda(|)$ and the revenue collected from all agents is necessarily equal to the one that is obtained when taxes are paid according to $\tau_\lambda(|)$ and larger than the revenue collected from all agents when tax liabilities are dictated with the optimal tax function, $\tau_E(|)$ Therefore, we can expect, relying on an intuitive notion of the mean value theorem[5], that there exists a value of y for which he revenue collected from all agents, who face $\tau_y(|)$, is the same as the revenue collected from agents when agents face the optimal tax function, $\tau_E(|)$ Let us denote such a value of y with $\bar{y}_R$.

Let us now assume that agents' tax liabilities are determined by $\tau_{\bar{y}_R}(|)$. Furthermore, let the labor supply choice, given tax function $\tau_{\bar{y}_R}(|)$, of an agent whose productivity is equal to a be denoted with L_a^{**} and the corresponding income earned with $y_a^{**} = aL_a^{**}$, and let the level of revenue collected be equal to

$$R_a^*(\lambda, \bar{y}_R) = \tau_{\bar{y}_R}(y_a^*). \tag{27}$$

Note that given the choice of $\bar{y}_R$ the revenue collected with $\tau_{\bar{y}_R}(|)$ must be the same as the revenue collected with the original optimal tax function, $\tau_E(|)$ i.e., we must have

$$\int_{a_L}^{a_H} R_a^*(\lambda, \bar{y}_R) f(a)da = \int_{a_L}^{a_H} \tau_{\bar{y}_R}(y_a^*) f(a)da = \int_{a_L}^{a_H} R_a^* f(a)da = R. \tag{28}$$

[4] At this stage, we make this assumption to preserve clarity of the exposition. We realize that this assumption needs not be satisfied. We consider a more realistic case in the online Appendix A.

[5] We provide a rigorous argument in the online Appendix B.

Furthermore, the realized utility in this case is given by

$$U_a^* = h\left(y_a^* - R_a^*(\lambda, \overline{y}_R) - \frac{1}{2}\left(\frac{y_a^*}{a}\right)^2\right). \tag{29}$$

Finally, the realized value of the objective functional, the dead weight burden, can be expressed as

$$DWB^{**} = \int_{a_L}^{a_H} \frac{1}{2}\left(\frac{a^2 - y_a^{**}}{a}\right)^2 f(a)da. \tag{30}$$

Observe that agents can be now split into two categories. Those who remain at their original choices when $\tau_\lambda(|)$ is replaced with $\tau_{\overline{y}_R}(|)$ and those who alter their behavior. Specifically, when the behavior is not changed, then by definition, we have $ya^{**} = ya^*$, and consequently it must be

$$\frac{1}{2}\left(\frac{a^2 - y_a^{**}}{a}\right)^2 = \frac{1}{2}\left(\frac{a^2 - y_a^*}{a}\right)^2. \tag{31}$$

It is necessary to consider two separate cases when the behavior is affected. First note that agents who initially chose incomes below $\overline{y}_R$ and now choose incomes above $\overline{y}_R$ pay more in taxes, i.e.,

$$R_a^{**}(\lambda, \overline{y}_R) = \tau_{\overline{y}_R}(y_a^{**}) > \tau_\lambda(y_a^*) = R_a^* + \lambda. \tag{32}$$

Furthermore, their previous choices remain available, so invoking the revealed preference argument we can write

$$h\left(y_a^{**} - R_a^{**}(\lambda, \overline{y}_R) - \frac{1}{2}\left(\frac{y_a^{**}}{a}\right)^2\right) \geqslant h\left(y_a^* - R_a^* - \lambda - \frac{1}{2}\left(\frac{y_a^*}{a}\right)^2\right) \tag{33}$$

which can be simplified to

$$y_a^{**} - R_a^{**}(\lambda, \overline{y}_R) - \frac{1}{2}\left(\frac{y_a^{**}}{a}\right)^2 \geqslant y_a^* - R_a^* - \lambda - \frac{1}{2}\left(\frac{y_a^*}{a}\right)^2. \tag{34}$$

In turn, by adding up (32) and (34), we can establish the following

$$y_a^{**} - \frac{1}{2}\left(\frac{y_a^{**}}{a}\right)^2 > y_a^* - \frac{1}{2}\left(\frac{y_a^*}{a}\right)^2. \tag{35}$$

which naturally implies that

$$\frac{1}{2}\left(\frac{a^2 - y_a^*}{a}\right)^2 > \frac{1}{2}\left(\frac{a^2 - y_a^{**}}{a}\right)^2. \tag{36}$$

The situation is slightly more complicated in the case of agents who initially chose incomes above $\overline{y}_R$. First of all, note that such agents would never switch to levels of income below $\overline{y}_R$, as those choices were available to them originally, and they chose incomes above $\overline{y}_R$. Moreover, now, they can actually pay less in taxes if they remain above $\overline{y}_R$, so switching to income levels below $\overline{y}_R$ is surely suboptimal. Furthermore, remaining above $\overline{y}_R$ entails, first of all, a lower amount of tax liabilities, and secondly, it implies that agents face a marginal tax of zero. Consequently, we can conclude that agents who were initially above $\overline{y}_R$ remain in the range of incomes above $\overline{y}_R$ and, in fact, do change their behavior in response to zero marginal taxes. In other words, the new labor supply choices coincide with the values of labor supply that would be chosen if taxes were lump sum. Accordingly, we must have

$$h\left(y_a^{**}-R_a^*-\lambda-\frac{1}{2}\left(\frac{y_a^{**}}{a}\right)^2\right)>h\left(y_a^*-R_a^*-\lambda-\frac{1}{2}\left(\frac{y_a^*}{a}\right)^2\right). \tag{37}$$

as supplying a given level of revenue, $Ra^* + \lambda$, is more efficient when marginal taxes are zero rather than positive. Equation (37) reduces to

$$y_a^{**}-\frac{1}{2}\left(\frac{y_a^{**}}{a}\right)^2>y_a^*-\frac{1}{2}\left(\frac{y_a^*}{a}\right)^2. \tag{38}$$

and further to

$$\frac{1}{2}\left(\frac{a^2-y_a^*}{a}\right)^2>\frac{1}{2}\left(\frac{a^2-y_a^{**}}{a}\right)^2. \tag{39}$$

Combining (31), (36), and (38), we can be sure that the following must be true

$$\forall a\in[a_L,a_H]\left|\frac{1}{2}\left(\frac{a^2-y_a^*}{a}\right)^2\geqslant\frac{1}{2}\left(\frac{a^2-y_a^{**}}{a}\right)^2,\right. \tag{40}$$

with a strict inequality on a non-degenerate set. Furthermore, the collection of inequalities (40) implies that

$$\int_{a_L}^{a_H}\frac{1}{2}\left(\frac{a^2-y_a^*}{a}\right)^2f(a)da>\int_{a_L}^{a_H}\frac{1}{2}\left(\frac{a^2-y_a^{**}}{a}\right)^2f(a)da. \tag{41}$$

Observe that by replacing the efficient tax function $\tau_E(|)$ with $\tau_\lambda(|)=\tau_E(|)+\lambda$ for points actually chosen, we increase the revenue collected, and we do not affect individual choices, i.e., we do not affect efficiency. Furthermore, by replacing $\tau_\lambda(|)=\tau_E(|)+\lambda$ with $\tau_{\overline{y}_R}(|)$ we enhance efficiency and reduce the revenue collected to the level obtained with $\tau_E(|)$ Clearly, as we replace $\tau_E(|)$ with $\tau_{\overline{y}_R}(|)$ we improve efficiency without compromising revenue. In other words, it is possible to collect, in a more efficient way, the same amount of revenue as collected with $\tau_E(|)$.

We can restate the argument formally, as we can express the original, when agents face $\tau_E(|)$ value of the dead weight burden, equation (18), as

$$DWB^*=\int_{a_L}^{a_H}\frac{1}{2}\left(\frac{a^2-y_a^*}{a}\right)^2f(a)da. \tag{42}$$

Similarly, the dead weight burden when agents face $\tau_{\overline{y}_R}(|)$ is given by, equation (30) − restated below,

$$DWB^{**} = \int_{a_L}^{a_H} \frac{1}{2}\left(\frac{a^2 - y_a^{**}}{a}\right)^2 f(a)da. \tag{43}$$

However, given (41), we can easily establish that

$$DWB^{**} < DWB^{*}. \tag{44}$$

Therefore, we must conclude that the original function, $\tau_E(|)$, cannot be efficient, as by replacing $\tau_E(|)$ with $\tau_{\overline{y}_R}(|)$ we not only preserve revenue, but also enhance efficiency since we lower the value of the objective functional expressed with (14). In other words, a given tax function cannot be efficient unless it is flat in a neighborhood of the highest income earned. Consequently, we can state that a given tax function cannot be efficient unless it is of the shape of $\tau_{\overline{y}_R}(|)$ to start with. However, by originating at y_H and moving to the left, we can repeat the above argument starting from any point y that is actually chosen and at which the efficient tax function stops being flat. Consequently, we can argue[6] that any efficient tax function must be a step function at least over an non-degenerate set of income levels.

To reiterate, we can state that it turns out that the class of functions implicitly considered plausible by Saez (1999) – who restricts his attention to differentiable functions $|-|$ can be too limited. Specifically, we can apply Saez's approach even when the distribution of skills is bounded. Let $\tau_S(|)$ be the efficient tax function obtained with Saez's method. Naturally, we can always apply our procedure[7] to $\tau_S(|)$. First we shift it up by λ, small enough, and then we flatten its right hand tail to ensure revenue neutrality. Naturally, by doing so, we obviously enhance efficiency, i.e., we make the value of the total dead weight burden smaller. In other words, it is possible to improve[8] upon Saez's solution if one is willing to extend the class of admissible tax functions and, in particular, consider step functions as admissible.

Furthermore, Saez (1999) argues that his dead weight burden minimization problem is equivalent to an optimal taxation problem with properly defined social welfare weights. However, this signals a possibility that solutions to some of the optimal taxation problems can be improved upon by extending the class of admissible functions. Specifically, by extending the class of admissible functions to functions that are not necessarily differentiable everywhere we can improve upon the solution of Saez in the context of efficiency. Furthermore,

[6] We provide precise reasoning in the online Appendix C.

[7] Appendix E considers the case when some agents choose not to work when they face $\tau_S(|)$.

[8] Note that we initially assumed that the revenue requirement, R, was sufficiently high to exclude the feasibility of a trivial solution in the form of a uniform tax of $\dfrac{R}{\int_{a_L}^{a_H} f(a)da}$. Consequently, our improvement is applicable in cases, considered of interest by Saez (1999), when some agents are not able to pay a uniform poll tax.

given the equivalence observed by Saez it may be possible to improve upon some solutions to optimal taxation problems if one allows for functions that are not necessarily differentiable everywhere.

4. PREFERENCES FOR REDISTRIBUTION

In his original contribution Mirrlees (1971) noted that policy makers apart from being interested in efficiency issues can also take into account equity considerations. In this paper, we adopt Mirrlees's approach, but we do not assume that the preferences of policy makers are defined over utility levels, but rather over the degree of inefficiency created by tax policies. In other words, in this paper we assume that policy makers exhibit aversion towards inequality with regard to the tax incidence, i.e., policy makers in our context are interested in ensuring that dead weight burden born by different agents is of similar magnitude. We choose to follow this approach as we believe that it complements Mirrlees's finding and moreover it is based on a more natural and meaningful measure such as dead weight loss rather than on a measure that is not directly comparable, such as individual utility. We may want to add that our results would not be affected if one were to consider just pre-tax income as a reference point.

As previously, let us assume that given tax function $\tau(\,)$ the problem of a given economic agent can be stated as

$$\max_{\{y\}} U = h\left(y - \tau(y) - \frac{1}{2}\left(\frac{y}{a}\right)^2\right). \tag{45}$$

Denoting with *ya* the optimal choice we can express the value of the dead weight burden as

$$DWB_a = \frac{1}{2}\left(\frac{a^2 - y_a}{a}\right)^2. \tag{46}$$

We can now define the problem of a policy maker as follows

$$\min_{\{r(\,|\,)\}} W = \int_{a_L}^{a_H} G(DWB_a) f(a)\,da \tag{47}$$

subject to

$$R = \int_{a_L}^{a_H} \tau(y_a) f(a)\,da \tag{48}$$

and

$$\forall a \in [a_L, a_H] \,|\, h'\left(y_a - \tau(y_a) - \frac{1}{2}\left(\frac{y_a}{a}\right)^2\right)\left(1 - \tau'(y_a) - \frac{y_a}{a^2}\right) = 0. \tag{49}$$

Function $G(\,|\,)$ captures the preferences of the policy maker. Following Mirrlees, we assume that $G(\,|\,)$ is increasing and convex[9], even though for our

[9] Note that we are interested in minimization of the dead weight burden. Consequently, working a convex rather than concave function $G(\,|\,)$ appears to be more proper if one intends to capture aversion towards inequality.

purposes only the former condition is relevant. Observe that our formulation of the problem is different than that proposed by Mirrlees. Specifically, in our case $G(|)$ is assumed to be a function of the dead weight burden whereas Mirrlees assumes that policy makers are utilitarian. In other words in our case policy makers are assumed to exhibit aversion towards variations in the degree of inefficiency faced by economic agents rather than in the *level* of overall happiness as measured by utility functions, which themselves are not comparable across agents.

We propose this alternative formulation for a number of reasons. First of all, it appears that the literature has so far ignored the problem, which appears to be of relevance. Secondly, the problem as proposed by Mirrlees (1971) appears to have been tackled given the advances of Diamond (1998) and Saez (2001). Finally, our formulation, we believe, in addition to its novelty is more natural than that proposed by Mirrlees. Note that in Mirrlees's approach, one must accept the notion that utility levels are meaningful and more importantly that cross person comparison of utilities can be performed. Those implicit assumptions of Mirrlees are normally taken as given and are normally not questioned. However, formally they contradict some of the basic foundations of economic theory. In our formulation, on the other hand, those standard problems do not arise as dead weight burdens for all individuals are measured in common units, which makes ranking of outcomes and cross personal comparisons meaningful. In other words, the problem as stated is not only novel but it is also well defined and it does not contradict any of the basic assumptions of economic theory.

Let $\tau_G(|)$ be the solution, assuming that it exists, to the problem described with equations (47), (48), and (49). Furthermore, let ya^* denote that the optimal choice of an agent whose productivity is equal to a when she faces tax function $\tau_G(|)$.

At this stage we cannot say anything meaningful about the shape of $\tau_G(|)$ apart from some basic observations[10]. However, let us assume that $\tau_G(|)$ actually exists and, for illustrative purposes, that its shape resembles the shape of optimal tax functions. Furthermore, recall that in the main body of the paper we focus on the case when all agents, given $\tau_G(|)$, strictly prefer to supply a positive amount of labor[11]. Under those assumptions the dead weight burden experienced by an agent whose productivity is equal to a can be expressed as

$$DWB_a^* = \frac{1}{2}\left(\frac{a^2 - y_a^*}{a}\right)^2 \tag{50}$$

and the corresponding value of the objective functional is given by

$$W^* = \int_{a_L}^{a_H} G\left(\frac{1}{2}\left(\frac{a^2 - y_a^*}{a}\right)^2\right) f(a)da. \tag{51}$$

[10] It is straightforward to establish that in this case just as it is in the case of optimal taxation the marginal tax paid by the most talented individual should be zero.

[11] Again, we consider the case when some agents are idle in the online Appendix E.

Let us now consider familiar variations of $\tau_G(|)$. First let us shift $\tau_G(|)$ over the relevant range up by λ, i.e., let us consider $\tau_\lambda(y)\ |=|\ \tau_G(y)\ +\ |\lambda|$ for values of y that are actually chosen[12]. Note that such a transformation has no bearing on individual behavior for λ sufficiently small. Furthermore, let us replace $\tau_\lambda(|)$ with a tax function, $\tau_y(|)$, that coincides with $\tau_\lambda(|)$ up to a given level of income, y, and then it is flat afterwards. Recall that $\tau_\lambda(|)$ brings more revenue than $\tau_G(|)$ and that we can find such a value, $\bar{y}_R$, see the discussion in the online Appendix *B*, that the level of revenue collected with $\tau_{\bar{y}_R}(|)$ is the same as the level of revenue collected with $\tau_G(|)$. Formally, we can state that there is $\bar{y}_R$ such that $\tau_{\bar{y}_R}(|)$ is feasible for the problem described with (47), (48), and (49), which can be stated as

$$\int_{a_L}^{a_H} \tau_G(y_a^*) f(a)da = \int_{a_L}^{a_H} \tau_{\bar{y}_R}(y_a^{**}) f(a)da = R, \tag{52}$$

where ya^{**} denotes the level of income earned by an agent whose productivity is equal to a when she faces tax function $\tau_{\bar{y}_R}(|)$

As argued in the previous section, the dead weight burden experienced by an individual when she faces $\tau_{\bar{y}_R}(|)$ is never higher than the level of dead weight burden when she faces $\tau_G(|)$. Consequently,

$$\forall a \in [a_L, a_H] \,|\, \frac{1}{2}\left(\frac{a^2 - y_a^*}{a}\right)^2 \geqslant \frac{1}{2}\left(\frac{a^2 - y_a^{**}}{a}\right)^2, \tag{53}$$

with a strict inequality on a non-degenerate set.

Furthermore, note that by assumption $G(|)$ is an increasing function. Consequently, given series of inequalities (53) we can write the following

$$\forall a \in [a_L, a_H] \,|\, G\left(\frac{1}{2}((a^2 - y_a^*)/a)^2\right) \geqslant G\left(\frac{1}{2}\left(\frac{a^2 - y_a^{**}}{a}\right)^2\right), \tag{54}$$

which implies, as the inequality in (54) strict for a non-degenerate set of a, that the following holds

$$W^* = \int_{a_L}^{a_H} G\left(\frac{1}{2}\left(\frac{a^2 - y_a^*}{a}\right)^2\right) f(a)da > \int_{a_L}^{a_H} G\left(\frac{1}{2}\left(\frac{a^2 - y_a^{**}}{a}\right)^2\right) f(a)da = W^{**} \tag{55}$$

Naturally, inequality (55) together with the feasibility condition (52) imply that $\tau_G(|)$ cannot be a solution to the problem described with (47), (48), and (49) unless it is locally flat in a neighborhood of the highest income earned for otherwise it could be possible to lower the value of the objective functional by flattening $\tau_G(|)$ at the high end.

The solution to the problem described with (47), (48), and (49) is again particularly simple. It turns out that allowing the policy maker to display aversion towards inequality does not change the solution in the qualitative sense. It is still the case that the solution takes the form of a step function. In other words, the

[12] We can assume that $\tau_\lambda(y) = y$ everywhere else.

preferences of the policy maker do not affect the form of the solution, but naturally determine its precise shape.

5. CONCLUSIONS

In this paper, we focus on the efficiency of taxation. Specifically, we modify the optimal taxation problem of Mirrlees (1971) by assuming that the preferences of policy makers are defined over the magnitude of distortions created by tax systems rather than the overall level of utility. In particular, we choose to measure distortions using the most natural metric, i.e., the dead weight burden imposed on individual agents by tax systems. Furthermore, we allow policy makers to exhibit aversion towards inequality, i.e., we assume that policy makers value outcomes that are characterized by a lesser dispersion of distortions.

We consider our approach and our results to be complementary to those studied and derived in the standard optimal taxation literature. At the technical level, our modification amounts to a change in the form of the objective functional. The change that we propose is a very natural one and it allows us to extend the optimal taxation literature in a meaningful manner. Specifically, by assuming that the preferences of policy makers are defined over the magnitudes of dead weight burden rather than over the utility levels, we eliminate the most profound criticism formulated with regard to the optimal taxation approach. Our specification and focus on dead weight burden rather than the level of utility makes comparison of utilities meaningful and allows for ranking of the values of the objective functional in a manner that is deeply rooted and consistent with basic economic theory. Furthermore, in our case the preferences of the policy maker are defined over concepts measured in common and comparable units making comparisons across individuals informative. Consequently, the extension that we propose is not only novel, but, in fact, it can be justified by fundamental assumptions of economic theory and not just by the applicability of a given method that facilitates the solution of the problem at hand.

In the paper, at the technical level, we work, following Diamond (1998) and Saez (2001), with a utility function that is essentially of a quasi-linear form, and we show, contrary to the findings of Saez (1999), that tax functions that ensure efficiency are of particularly simple form. Specifically, we show that efficient tax functions, if they exist, are locally flat implying generically zero marginal tax rates. In addition, we prove that our findings hold even when policy makers exhibit aversion towards inequality.

We view our contribution to be of value at a number of levels. First of all, we propose a framework that is, unlike the optimal taxation literature, fully compatible with basic economic theory. Secondly, while the approach of Mirrlees (1971) has become dominant it is not the only one. Recently, I. Werning (2007), and Saez and Stantcheva (2012) have noted the limitations of the approach of Mirrlees and have proposed alternative formulations of the optimal taxation problems. In this paper, we follow a similar path; we propose an alternative, which is very natural, and leads to new and meaningful results.

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ABSTRACT

We solve an often neglected problem of efficient taxation where the policy maker is interested in minimizing the dead weight burden imposed on the society by labor income taxes. Specifically, we argue that the efficient tax function assumes the form of a step function. Furthermore, we allow the policy maker to exhibit aversion toward inequality and show that the key result holds, i.e., that the tax function that ensures efficiency is locally flat.

Keywords: efficient taxation, dead weight burden, step function, zero marginal taxes, quasi-linearity.

JEL Classification: H21

PROBLEM EFEKTYWNEGO OPODATKOWANIA

STRESZCZENIE

W artykule przedstawiono rozwiązanie problemu efektywnego opodatkowania – prowadzącego do minimalizacji straty pustej, nałożonej na społeczeństwo. W szczególności w artykule wykazano, iż efektywna funkcja opodatkowania przyjmuje postać funkcji schodkowej. Dodatkowo pozwalamy na modyfikacje podstawowego problemu i pokazujemy, że kluczowy rezultat, iż efektywna funkcja opodatkowania przyjmuje postać funkcji schodkowej pozostaje w mocy.

Słowa kluczowe: efektywne opodatkowanie, strata pusta, funkcja schodkowa, zerowa krańcowa stopa opodatkowania, quasi-liniowość.